Black holes, entropy and information

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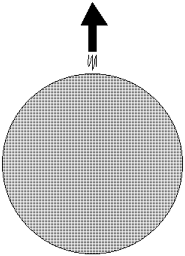
Black hole basics: History

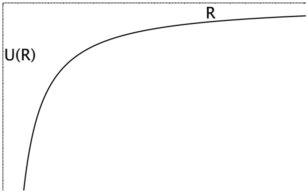
Laplace (1795), Schwarzschild (1915), Oppenheimer (1939), Wheeler (1967).

Laplace: when is escape velocity equal to the speed of light?



$$\frac{v^2}{R} \sim \frac{GM}{R^2} \quad \rightarrow \quad R \sim GM/c^2$$

Black hole basics: History



Energy of two particle system:

$$E = 2Mc^2 + U(R) = 2Mc^2 - G\frac{MM}{R}$$

$$E = 0 : \; R \sim GM/c^2$$

If we allow R less than the Schwarzschild radius, there would be a vacuum instability!

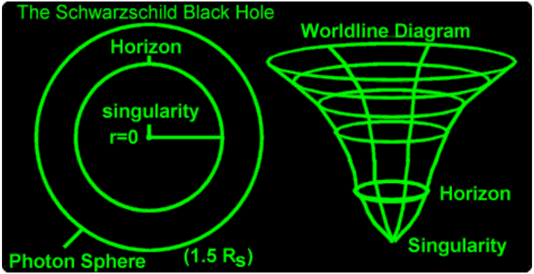
Black hole basics: History

Schwarzschild solution in general relativity (1915).

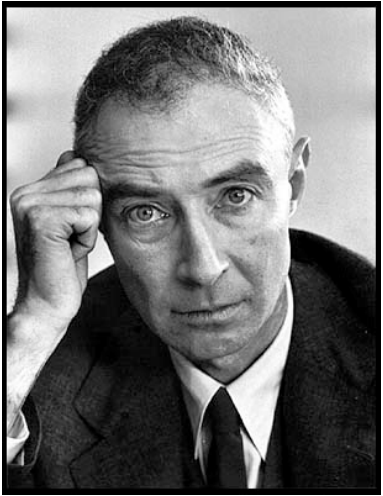
$$ds^2 = (1 - r/R)\, dt^2 - (1 - r/R)^{-1}\, dr^2 - r^2 d\Omega^2$$

$$R = 2GM$$

(Henceforth, use natural units in which $G = \hbar = c = 1$.)



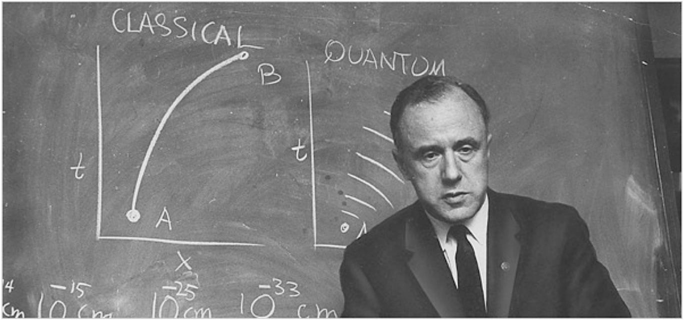
Black hole basics: History



Oppenheimer: gravitational collapse, stellar evolution (1939).

Black hole basics: History

Wheeler: coined the term “black hole” (1967). Soviets: “frozen star”?



Black holes: big and small

$$R \sim M$$

Radius in Planck lengths (10^{-35} m) is roughly energy in Planck masses (10^{-5} g).

If ρ = average density, then $M \sim \rho R^3$,

$$\rho_{\max} \sim 1/R^2$$

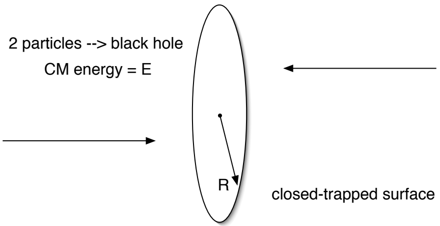
A galactic mass black hole (at formation) is less dense than water! Not so mysterious... or is it?

Black holes: big and small

$$R \sim M$$

Radius in Planck lengths (10^{-35} m) is roughly energy in Planck masses (10^{-5} g).

Small black holes: created in accelerators? $M > 10^{-5}$ g



Eardley and Giddings ; Hsu (2004)

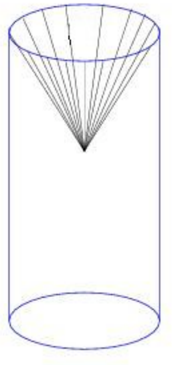
No Hair

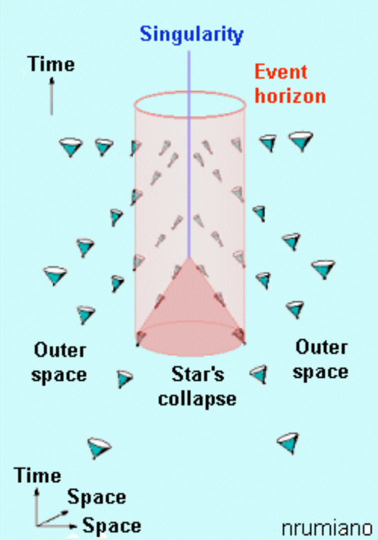
External metrics of classical black holes easily categorized.

No Hair theorems:

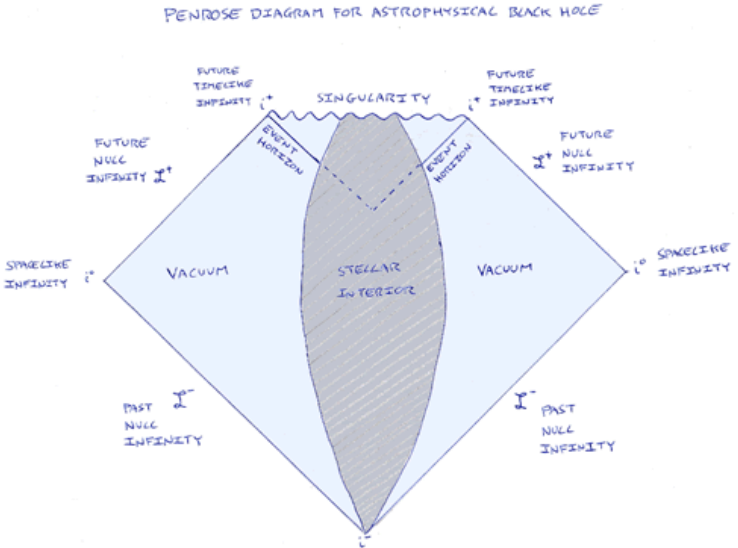
$$M,\ J,\ Q,\ \cdots$$

Lightcones and causality

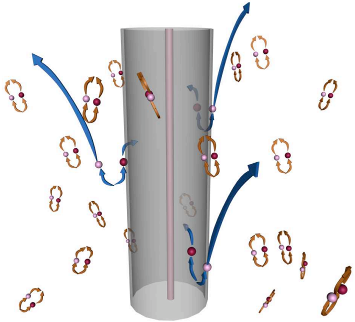




Penrose diagram: eternal hole



Hawking radiation

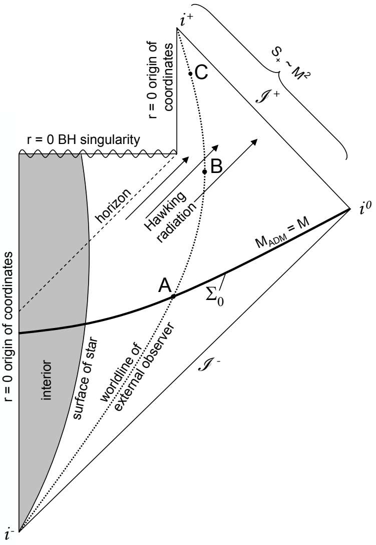


- Quantum fluctuations just outside the horizon lead to radiation. (e.g., e^+e^- pairs)

- Temperature is $T \sim 1/R \sim 1/M$ (very low)

- Lifetime of hole is $\tau \sim M^3$ (very long)

Penrose diagram: evaporating hole



Black holes are not eternal.

Hawking radiation originates outside the horizon.

Black hole entropy

With temperature comes entropy: $S_{\mathrm{BH}} = A/4$

Entropy given by area in Planck units. One bit per Planck area.

Bekenstein-Hawking: $T \sim R^{-1}$, $R \sim M$

$$S_{\mathrm{BH}} \sim \int \frac{dQ}{T} \sim \int R\, dM \sim R^2$$

String theory microstate counting: Strominger and Vafa

The meaning of entropy

Entropy = log of number of microstates consistent with some macro condition.

$$S \sim \ln \{\# \text{ of microstates } N\}$$

Typically, entropy is *extensive*:

$$\ln (c^V) = V \ln c$$

The dimensionality of the Hilbert space describing a volume V is $\dim\mathcal{H} = c^V$ ($c = 2$ for qubit). $S \sim$ number of d.o.f.

- coarse graining
- loss of information

Entropy bound from black holes

Any ordinary system can be converted (smashed!) into a black hole by adding energy (increasing its density).

Its entropy before becoming a black hole must be less than that of the resulting hole.

We seem to have deduced an upper bound on the entropy of any physical system:

$$S < A \sim M^2$$

Holography, dimensional reduction, extensivity?

'tHooft bound

Exclude states whose energies are so large that they would have already caused gravitational collapse: $E < R$ $(\rho < \rho_{\max})$

Compute entropy; dominated by thermal configurations at large V:

$$S \sim T^3 R^3 \quad , \quad E \sim T^4 R^3$$

$E < R$ then implies

$$T \sim R^{-1/2} \quad \rightarrow \quad S < R^{3/2} \sim A^{3/4}$$

$$S < A^{3/4}$$

Note: Black hole density decreases with size. For any constant density $E(R) > R$ for sufficiently large R!

’tHooft bound

These bounds are less confusing if we put the units back in.

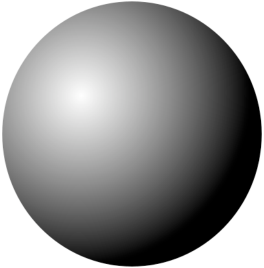
When we measure the area in Planck units (the Planck length is $10^{-35}m$) the upper bounds on S are numerically very large.

The expectation that S is extensive: $S \propto V$, only conflicts with the upper bounds for such large systems that gravity is non-negligible and in fact will force the system to become a black hole.

’tHooft bound

Ordinary matter satisfies $S < A^{3/4}$. What does this say about black holes, which have $S \sim A$? There is an entropy gap!

Black holes have much more entropy than ordinary matter configurations of the same size / energy.

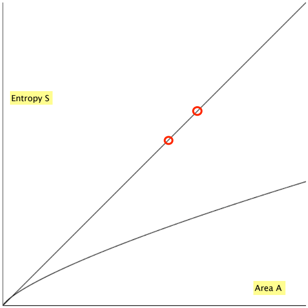


Ordinary matter: $\exp A^{3/4}$ possible states

Black hole: $\exp A$ possible states

Entropy gap

Black holes have much more entropy than ordinary matter configurations of the same size / energy.



The number of ways to form a given black hole is overwhelmingly dominated by simply adding some energy to a slightly smaller hole.

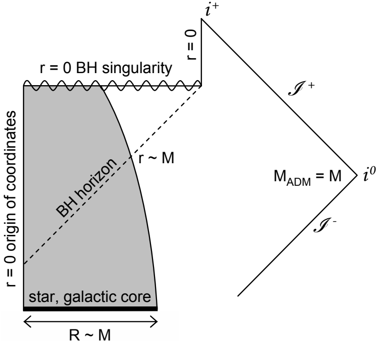
Black hole entropy

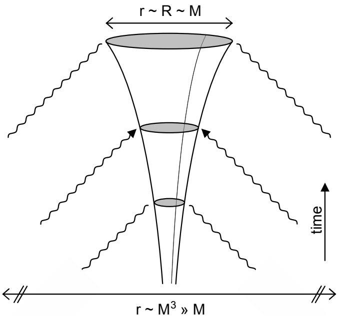
Ordinary star collapses to form black hole:

$$S < A^{3/4} \sim M^{3/2}$$

Existing hole slowly absorbs photons (time reversal of Hawking radiation): can maintain

$$S \sim A \sim M^2$$





Closing the entropy gap

Negative binding energy

Consider N particles of mass m. In GR there is negative binding energy $\equiv \Delta$. The total energy can be less than Nm:

$$M = Nm - \Delta$$

In fact, one can achieve

$$\frac{M}{Nm} = \frac{Nm - \Delta}{Nm} \ll 1 \quad .$$

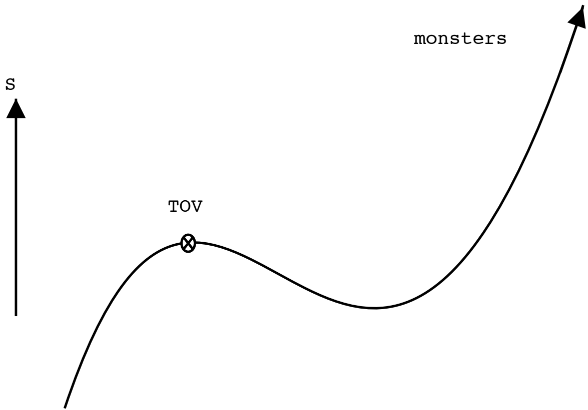
This suggests that entropy to mass ratios can be very high, for special configurations.

ADM mass M: gravitational mass as detected by asymptotic observers.

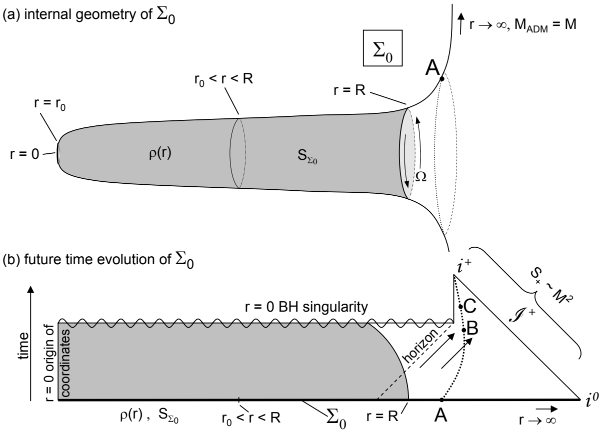
Entropy extrema

SH and D. Reeb 2006: Monsters! See also Sorkin, Wald and Zhang 1981.

Hold ADM mass M fixed, extremize entropy S.



Monster spacetime



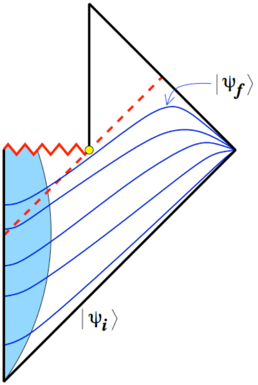
Monsters

Fill the "entropy gap" between $A^{3/4}$ and A scaling

But, at semi-classical level, no reason to exclude monsters with $S > A$: violate naive entropy bound, problems for holography and AdS/CFT.

New selection rules in quantum gravity?

Unitarity and the black hole information paradox



How does the future state of the universe ψ_f depend on the past state ψ_i?

Radiation is produced long after hole forms, in a region which is causally disconnected from the interior.

Almost all the radiation intersects a spacelike *nice-slice*; the No-Cloning theorem prevents information from existing at two places on a spacelike slice (i.e., simultaneously).

No cloning theorem

Theorem: It is impossible to create identical copies of an arbitrary unknown quantum state.

Proof: Suppose $\exists\, U$ such that

$$U\,|\psi\rangle \otimes |e\rangle = |\psi\rangle \otimes |\psi\rangle$$

and

$$U\,|\phi\rangle \otimes |e\rangle = |\phi\rangle \otimes |\phi\rangle$$

then

$$\langle\phi|\psi\rangle = \langle e| \otimes \langle\phi|\, U^\dagger U\,|\psi\rangle \otimes |e\rangle = \langle\phi| \otimes \langle\phi|\; |\psi\rangle \otimes |\psi\rangle = \langle\phi|\psi\rangle^2$$

which implies ψ and ϕ are either equal or orthogonal, so they cannot be arbitrary (generic) states. (Could also consider superpositions.)

Unitarity and the black hole information paradox

Possible solutions:

1) Unitarity:

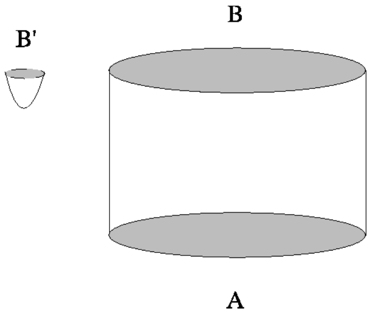
$$\psi_f = U\,\psi_i = e^{-iHt}\,\psi_i\,,$$

but how does the information get out? non-locality?

2) Remnants: black hole evaporates down to Planckian size, quasi-stable object. Remnant must be entropically superdense.

3) Apparent non-unitarity / baby universes: information appears to be destroyed, but ends up somewhere else. Multiverse obeys unitary evolution.

Baby universes?



Spacetime topology change.

Information ends up in baby universe.

Apparent non-unitarity in parent-universe, but entire multiverse evolution is unitary.

What is the entropy of the universe?

Let N = number of quantum states which are consistent with the macroscopic information we have about the universe.

Then the entropy of universe is $S = \ln N$.

A measure of our uncertainty or ignorance about the precise state of the universe.

Also, if $\mathcal{H}$ is the Hilbert space describing the entire universe, then $dim\,\mathcal{H} = N = \exp S$.

Is S dominated by black holes?

Penrose ; Frampton, Hsu, Kephart and Reeb 2008

What is the entropy of the universe?

objects	entropy	energy
10^{22} stars	10^{79}	$\Omega_{\text{stars}} \sim 10^{-3}$
relic neutrinos	10^{88}	$\Omega_{\nu} \sim 10^{-5}$
stellar heated dust	10^{86}	$\Omega_{\text{dust}} \sim 10^{-3}$
CMB photons	10^{88}	$\Omega_{\text{CMB}} \sim 10^{-5}$
relic gravitons	10^{86}	$\Omega_{\text{grav}} \sim 10^{-6}$
stellar BHs	10^{97}	$\Omega_{\text{SBH}} \sim 10^{-5}$
single supermassive BH	10^{91}	$10^{7} M_{\odot}$
$10^{11} \times 10^{7} M_{\odot}$ SMBH	10^{102}	$\Omega_{\text{SMBH}} \sim 10^{-5}$
holographic upper bound	10^{123}	$\Omega = 1$

Table: Entropies and energies for various systems (using area entropy for black holes).

What is the entropy of the universe?

- If black hole evaporation is unitary, collapse to a black hole does not add any additional microstates to the accessible Hilbert space of the universe, $\mathcal{H}$.

- The entropy, in a fundamental sense, does not go up. The future radiation state ψ_f is fully determined by the initial state ψ_i. (However, for an observer with limited ability to differentiate between Hawking radiation states, there is an effective increase in entropy.)

- In the case of unitary evaporation, the dominant contribution to our uncertainty about the precise state of the universe is not black holes. Instead, it is our ignorance of the precise quantum state of CMB photons and neutrinos.

Black holes

• Defined by their effect on the causal structure of spacetime: existence of a horizon.

• The densest objects in the universe – but not necessarily dense in absolute terms (at formation).

• Have entropy, emit thermal radiation (weakly), eventually evaporate...

• Still full of mysteries for theoreticians!