# The Motion of Test Particles and Cosmological Interpretations: The Role of MOND

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## Abstract:

Throughout history, observations of the motions of objects in the Universe have provided the foundation for various cosmological models. In many cases, the invoked causes of the observed motion appeal to mysterious elements. Indeed, the very first test motion was that of the retrograde motion of Mars which lead to a required epicycle to save the model (e.g. Ptolemy's unmoving Earth. By the early 1840s, from approximately 50 years of orbital data (since its 1789 discovery) it was apparent that Uranus was disobeying the Newtonian rules in its orbit and speculation mounted that a "large unseen mass" was perturbing the orbit. Using Uranus as a test particle then yields the first notion of Dark Matter (DM). Alas, it was not DM but merely Neptune, discovered in Sept 1846. By 1859 enough data had been gathered to reveal that Mercury is also not obeying Newtonian physics but rather revealing curved space time. The continuation of this history is now set in scales larger than the Solar System. Observations suggest two basic choices: a) gravity is fully understood and Newton's second law is invariant (except in very strong gravity) and observed motions on galactic scales require the existence of DM (a currently unproven "epicycle") or b) Newton's second law can be modified (e.g. MOND) in certain low acceleration scale environments. In this contribution we discuss the case for and against MOND on various scales and conclude that neither MOND nor our current cosmology ($\Lambda$CDM) consistently explain all observed phenomenon. In general, MOND works much better on small scales than $\Lambda$CDM but encounters difficulties on large scales. Moreover, the nature of the acoustic power spectrum of the CMB now pretty clearly shows that a fully baryonic Universe is ruled out, thus necessitating some DM component. But this should not diminish the consideration of MOND as its introduced acceleration scale; $a_o$ is fully consistent with the observed structural properties of galaxies in a way that the DM halo paradigm cannot match. Indeed, despite many attempts to falsify MOND, it has always come back from its proclaimed death to provide unique insights into the gravitational nature of galaxies, consistently raising the specter that our current understanding of gravity acting over large spatial scales may be flawed.

# I. Introduction

When measuring the space motion of any test particle there are two assumptions that can be made about that motion: A) the motion is caused by known forces and the test particle is a probe of the combined gravitational mass acting on it or B) the motion of the test particle is responding to an unknown potential which may depend on the actual position of the particle within that potential. Option A, of course, is the preferred option under the assumption that Newtonian force laws in combination with the space-time curvature under general relativity (GR) are valid at all locations in the universe. Under this assumption, test particles probe the mass distribution and most all astrophysical observations of test particle motion point to the necessity of dark matter (DM - gravitational masses that emit little or no light). Option B was first introduce in Milgrom (1983) who suggested that the operational signal of DM would be similar to a basic modification of Newton's second law such that F = ma becomes F $=ma_{eff}$ where the effective acceleration ($a_{eff}$) can be expressed as:

$$a_{eff} \sim \sqrt{g_n(1 + a_o)}$$

Where $g_n = \mathrm{GM/R}^2$ (the Newtonian acceleration scale) and $a_o$ is a constant. When $g_n >> a_o$ then standard Newtonian dynamics applies. When $g_n \sim a_o$ the acceleration becomes constant and independent of scale. This is modified Newtonian dynamics (MOND). MOND results in altering Newton's second law, F=ma in the sense of requiring less gravitational mass to produce the observed acceleration. Thus the "missing mass" problem turns into a "missing gravity" one. The scale of $a_o$ is approximately $10^{-11}$ the surface gravity of the Earth or, conveniently, ~ 1 angstrom/sec$^2$. While such a scale is orders of magnitude below that which can be achieved in a terrestrial laboratory, there are a variety of astrophysical systems (see Table 1) whose enclosed acceleration scale is of order $a_o$. Note also that F =ma can be operationally modified by stating that the inertial mass of the object depends on upon location within a gravitational potential and therefore m becomes $m_{eff}$. This view of MOND is especially troubling as it violates the equivalence principle by implying that there can be astrophysical environments in which the inertial mass of an object does not equal its gravitational mass. The "inertial modification" form of MOND has been extensively invested by Milgrom (1994) and Milgrom (2011a).

The most extensive review of MOND is that of Famaey and McGaugh (FM) (2012). Other reviews of MOND include Sanders (2008), Milgrom (2014, 2011b, 2002, 2001, 1994), Bekenstein (2006), and Bekenstein and Milgrom (1984). This present review, while borrowing from these other works, is intended to be written from a different perspective, a perspective of a neutral observer that has designed observations to directly falsify MOND only to be continuously thwarted. As discussed in Burigana etal (2009) and Bothun (2013), alternative paradigms are often strongly suppressed by "known" science, even though, in the case of modern cosmology the "known" science rests on the existence of dark matter (DM) and dark energy (DE) as necessary conditions with currently no **direct** evidence of the existence of either one. Under that boundary condition, the notion that MOND or MOND-like alternatives to the gravitational force law in fact exist, is not a ridiculous notion to entertain.

## II. Introduction:  Test particle motions throughout History

According to Ptolemy in the Almagest:

> *It is manifest to any observer that the [spherical] earth occupies the middle place of the cosmos, and that all weights move toward it.*

Under this directive, the observed retrograde motion of Mars (a test particle) requires the introduction of the epicycle to "save the phenomena" (Plato).  For our purposes, it is useful to broaden the definition of an epicycle to be this:

> ***Any invoked device whose physical nature is completely mysterious and questionable but whose existence is absolutely required to preserve the model***.

This use of definition allows our current cosmological necessities of DM and DE to be considered as the required epicycles in the same way that MOND could be considered as an epicycle.

Next, Tycho's precision measurements of Mars over a 20 year period allowed Kepler to realize it had an accumulated positional error of 8 arcminutes relative to the situation if its orbit was perfectly circular; the ellipse then replaces the perfect circle cosmology of Aristotle (and Copernicus).  Next, Uranus was discovered with naked eye telescopic observations in the year 1789.  By the early 1840s enough positional measurements had been made to realize that the orbit of Uranus was not a single ellipse.  It was then suspected that another nearby gravitating mass was causing these perturbations and in 1846 that perturbing mass was confirmed as Neptune.  Next, by 1859 it was widely observed that the orbit of Mercury did not conform to Newton's laws.  In analogy with Neptune, the presence of an unseen perturbing body (dubbed Vulcan) was suspected but quickly ruled out. Hence the motion of this particular test particle had revealed a fundamental flaw in the Newtonian Universe.  This flaw is corrected in GR in which gravity and gravitational acceleration are now properties of the local curvature of space time.

By the late 1970s it was observationally determined (Bosma 1981, Rubin etal 1982) that the orbital velocities of stars in the outer parts of galaxies were faster than expected, if the mass in disk galaxies were centrally concentrated, like the light was.   More specifically, the expected Keplerian fall of stellar or gas orbital velocity vs radius was not seen. To rectify this DM is introduced with the requirement that the dark matter distribution is much less centrally concentrated than the light distribution.  Figure 1 symbolizes the current conventional wisdom regarding the structure of galaxies in which the luminous baryonic component sits in a much extended invisible halo of DM. This DM halo then contributes 90-99% of the total mass of any galaxy system, and yet there is no direct evidence for its existence at this time (as well as currently).

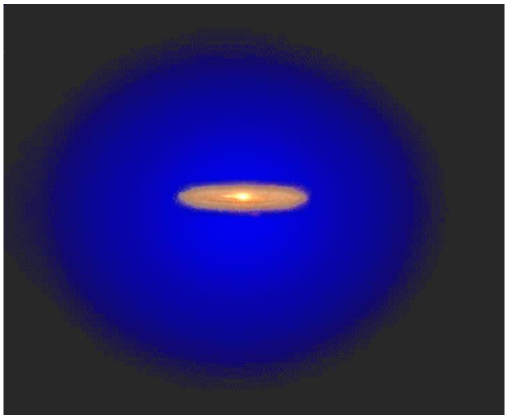

**Figure 1: small scale baryonic disk (orange) embedded in large scale DM halo (blue)**

Prior to this discover of "flat rotation curves" in spiral galaxies, there were other astrophysical indications of "missing mass" (the original terminology used to describe situations where test particles seemed to be moving faster than they should.) In a highly flattened rotating stellar system, the vertical (z) density distribution is given by Poisson's equation:

$$\frac{\partial^2 \phi}{\partial z^2} = 4\pi G\rho.$$

As the density increases, the z-coordinate sees a larger derivative in the potential which means it experiences a larger gravitational restoring force in that direction. Since stars are not gravitationally escaping in the vertical direction from the rotating disk of our Galaxy, Oort (1932) applied the collisional Boltzmann equation in combination with the equation for mass continuity:

$$\frac{\partial}{\partial_z}\left[\left(\frac{1}{v}\frac{\partial(v < \sigma_z >^2)}{\partial z}\right)\right] = -4\pi G\rho$$

To derive, using the measured vertical velocity dispersion of nearby stars, that the mass density in the solar neighborhood was 0.15 $M_\odot$ pc$^{-3}$. This estimate was revised upwards to 0.18—0.21 $M_\odot$ pc$^{-3}$ by Bahcall (1986). Observational accounting of the baryonic material (stars, gas, dust, stellar remnants) in the solar neighborhood revealed that 0.05 – 0.06 $M_\odot$ pc$^{-3}$ of material (mass) was "missing". The later discovery of the "thick disk" (Gilmore and Reid 1983) structural component of our galaxy removed the requirement for any "missing mass".

At around the same time as Oort's discovery of "missing mass" on the local galactic disk scale, Zwicky (1933) measured the velocity dispersion of galaxies in the nearby Cancer cluster of galaxies, to conclude that the binding mass of the cluster could not be in the form of the luminous galaxy members by at least a factor of 10, under the assumption that a cluster of galaxies was a self-gravitating bound isothermal sphere. Subsequence observations by Bothun etal 1983 revealed the existence of substantial sub-structure within this cluster and showed that, in fact, it was an "unbound" collection of small galaxy groups and most of the "missing mass" discrepancy was again removed.

These historical landmarks are important for showing a rather continuous chain of erroneous structural assumptions that map into observed motions of test particles to conclude that something is "missing". These landmarks serve as a good object lesson for the consideration of MOND. Analogous to how the anomalous motion of Mercury lead to a strong refinement on the nature of space-time - perhaps the observed motions that require DM may illustrate something more fundamental – that is, the nature of gravitational acceleration may be changing in certain environments. In the absence of the direct detection of DM, this alternative remains viable.

## III. Astrophysical Environments at the $a_o$ Scale:

The MOND acceleration scale is ~ 1 Å $s^{-2}$. A troubling aspect of MOND is the lack of any conceptual or theoretical framework for this acceleration scale. While the value of $a_o$ is fairly well determined empirically (see McGaugh 2004, McGaugh 2011, Walker & Loeb 2014), its appearance as a central scale factor that governs the properties of galaxies seems rather ad hoc.

A useful parameterization defines a critical length $R_t = \sqrt{GM/a_o}$ for any astrophysical systems. Example astrophysical systems are summarized in Table 1

Table 1: Astrophysical Environments where $GM/R^2 \sim a_o$

| System | M ($M_\odot$) | $R_t$ (light years) | Comments |
|---|---|---|---|
| Solar System | 1 | .05 | Objects at this radius can still be bound to the Sun but are subject to perturbation the periodic (every 50 million years or so) passage of giant molecular clouds |
| Globular Cluster | $10^6$ | 50 | Stars at this radius can be weakly bound to globular clusters but they also experience the mean tidal field of the Galaxy |
| Typical Galaxy | $10^{11}$ | 15,000 | This is less than the optical radius of a galaxy |
| Galaxy Cluster | $10^{14}$ | $1.5 \times 10^6$ | Typical radius of a cluster of galaxies |

The systems listed in Table 1 are all gravitationally bound astrophysical systems that exist within the MOND regime. Thus, $a_o$ systems do exist and within these systems there are numerous available test particles. As a consequence it should be observationally possible to experimentally rule out or confirm MOND as a credible alternative to DM. In fact, recent observations of stellar motions in the vicinity of the globular cluster NGC 2419 (Ibata etal 2011) have been used to rule out MOND but that has been well refuted by Sanders (2012). However, as noted in the table above, globular clusters are not good isolated systems to use kinematic evidence against MOND as member stars will feel both the globular cluster potential as well as some component of the Galactic potential. The cleanest kinematic system available for testing MOND would be galaxies with low values of $R_t$. These galaxies exist (see Section V) and will be referred to as LSMD (low surface mass density) galaxies.

## IV. A Gedanken

An unsettling aspect of MOND reveals itself through a simple thought experiment. Imagine that there exists only two masses in the Universe and that those masses are gravitationally bound. For simplicity let's consider the Sun and the Earth. Now suppose that Descartes Evil Genius shows up and starts to gradually pull the Earth away from the sun. Initially nothing will happen since as long as the Earth has an orbital radius < $R_t$ it remains in the Newtonian regime and its orbital dynamics are governed by Kepler's laws. As the Earth gets closer to $R_t$ Kepler's laws will increasingly no longer apply as the centripetal acceleration ($V^2/R$) will approach a constant value ($a_o$). Alternatively the gravitational mass of the Earth will begin to depart from its inertial mass so as to modify F=ma to $F=m_{eff}$ a. When the Earth gets towed well beyond $R_t$ it is now strictly a MONDian object and the entire physical nature of the Sun-Earth system has changed. Well, then, what exactly happens in the Newtonian to MOND transition region? Does the Earth enter into a superposition of a Newtonian state and a MOND state? Furthermore, would what happen if the Earth developed a very eccentric elliptical orbit such that at perihelion its distance is 10 $R_T$ and at aphelion its 0.1 $R_T$? This would imply a continuous transition in the difference between inertial mass and gravitational mass and this seems preposterous. While the above is clearly over simplistic and there have been serious attempts at constructing the interpolation function between the Newtonian and MOND regimes (see section 6.2 in FM 2012), the necessity of such interpolation physics can generally be viewed as inelegant.

## V. Attempts to Directly Falsify MOND

An observational fact not fully appreciated by the community is that all of the current evidence for anomalous motions and the corresponding need for DM to drive the observed acceleration actually come from environments which are at or near the MOND acceleration scale. As an example, in a typical galaxy a flat rotation curve (RC) appears only beyond the radius where the centripetal acceleration $V^2/R$ is less than $a_o$. Interior to the radius there is little evidence of anomalous motions (which would then provide a strong case against MOND). The long standing "missing mass" problem in clusters of galaxies could easily be a manifestation of MOND as most bound clusters have an enclosed acceleration scale close to $a_o$.

Evidence of DM on the scale of the Solar System (see Sanders 2006) would also rule out MOND. For instance, The "Pioneer anomaly" (the observed deceleration of the Pioneer 10 and 11 spacecraft, launched in 1972) has been used as evidence for the gravitational force low deviating from its strict inverse square dependence (Nieto and Turyshev 2004). However, at the time of the last received signal (2003) the space craft was located at a distance of 80 AU from the sun, a factor of 100 less than $R_t$ and therefore in an acceleration scale well above the MOND regime. The "anomaly" has also been suggested to be due to DM drag in the case of particle DM that is present in the Solar system (and everywhere else). The most likely explanation, however, involves a thermal recoil force generated by anisotropic emission of thermal radiation from the plutonium battery packs; an explanation which has nothing to do with either DM or MOND.

**Galaxy Rotation Curves**:

The discovery of LSMD galaxies in the mid-1980s (e.g. Bothun etal 1987) provided a natural laboratory to critically test MOND as some extreme members of that class have $GM/R^2 < a_o$ **at all radii**. Studying these systems has the added advantage that the exact form of the interpolation function between the MOND and Newtonian regimes is quite unimportant. In general, these galaxies are rich in gas and their dynamics could be obtained by observing the hydrogen emission line at 21-cm. McGaugh (1992) obtained rotation curves (RCs) of a fair sample of LSMD galaxies. I was McGaugh's thesis advisor and I helped design this experiment with the direct intention of falsifying MOND. Remember, that in the strict MOND paradigm, there is no DM and all the mass of a galaxy would be in the form of baryons. In contrast to "normal" galaxies, LSMD galaxies evolve at a slower rate in the sense that a) they are converting their gas content to stars at a slower rate on the billions of year's timescale, b) their stellar populations are generally metal poor and c) their molecular gas and overall dust contents are very low. As a consequence of this slower evolution, LSMD galaxies have significantly higher fractional gas masses and therefore the gas content is an important baryonic component and therefore accurate gas masses have to be determined. This is easily done from 21-cm fluxes and stellar masses could be estimated from the overall spectral energy distribution of the galaxy light. In astronomical terms, this is codified as the mass-to-light ratio (M/L) expressed in solar units. Older stellar populations have higher M/L values than younger ones and are redder (cooler) than younger populations. Thus measuring the radial "colors" of galaxies provides direct information on the local value of M/L for any stellar population.

The results of McGaugh (1992) were both disappointing and interesting. In brief, this initial attempt to directly falsify MOND failed miserably. Instead, many of the a priori predictions of MOND were **strongly** confirmed. MOND fits (where the gas+star =baryons provide the mass) to the RC data were better than DM fits with spherical DM halos. This result has since been confirmed with higher resolution data (Kuzio de Naray et al. 2009). Figure 2 (reproduced from FM 2012) shows example MOND fits (blue lines) to the RCs of LSMD galaxies. The x-axis is orbital distance (in units of kiloparsecs) and the Y axis is the observed circular (orbital) velocity at that distance. The black line represents the enclosed baryonic mass as a function of radius. In the MOND world all the mass is in baryons and the blue line represents the M/L scaled version of the black line. In the DM fits, the black line can be thought of as the amount of "missing mass" as a function of galactic radius. This is known as the "mass discrepancy". In all cases, in the DM model, galaxies are increasingly dominated by DM at large radii indicating that the baryonic mass fraction of galaxies is a function of galactic radius.

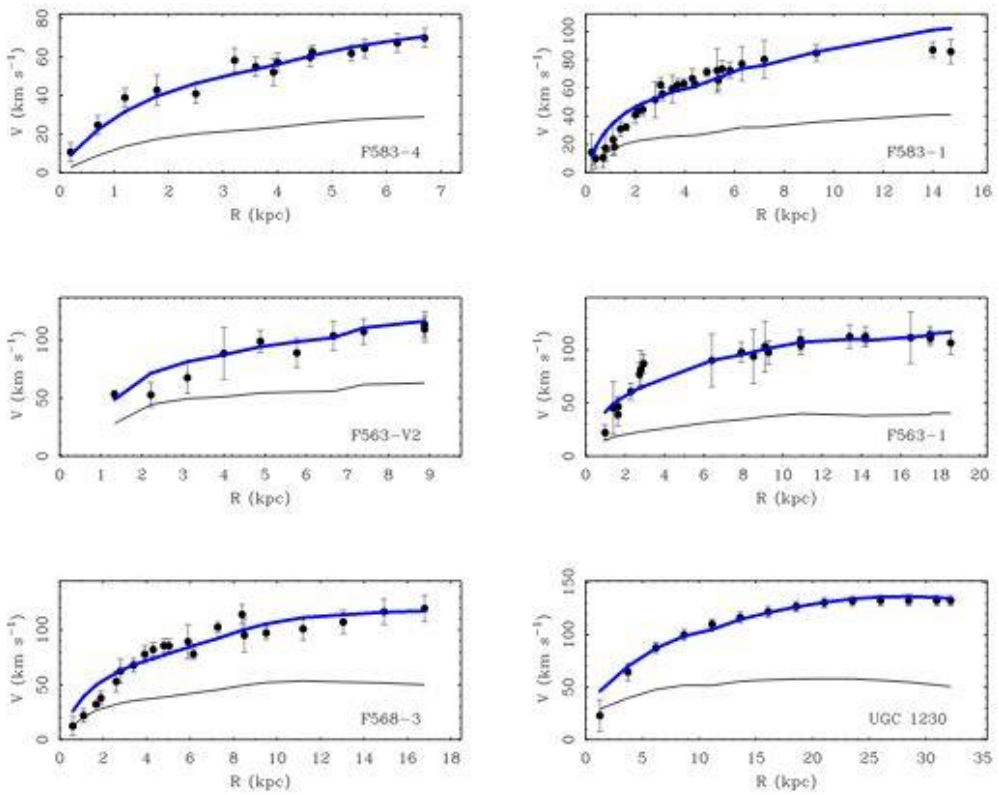

**Figure 2: MOND fits (blue line) to example LSDM galaxies obtained by scaling the black line by the mass-to-light ratio of the stellar population and the observed gas content as a function of radius.**

The difference between LSMD galaxies and normal galaxies is best revealed in Figure 3 (again adopted from FM 2012)

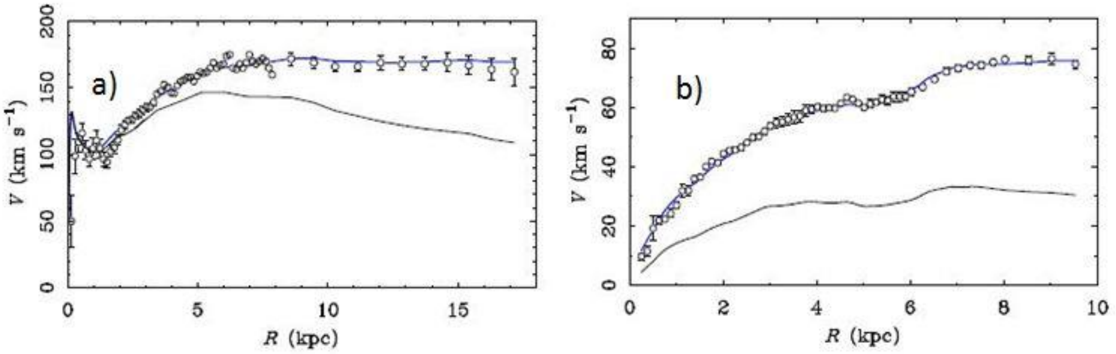

**Figure 3 Comparison of RCs for normal (panel a) vs LSMD galaxy (panel b)**

The galaxy in panel A (NGC 6946) is a typical galaxy like the Milk Way and in such galaxies their inner parts are dominated by baryons. There is no need to invoke DM as a parameter in their inner RC fit. Moreover, these inner regions have length scales < $R_t$ . For distance R > 10 kpc in panel A there is an increasing need for DM to make up the mass contribution and R > 10 kpc is beyond $R_t$. Of course, the MOND fit (blue line) to Panel A is a very good fit. Panel B is an example of another LSMD galaxy in which the actual features (bumps and wiggles) are very well represented by the scaled M/L MOND Fit with no need to appeal to DM as a mass component. Overall the flat rotation curves observed for galaxies can be fit as well with MOND as with DM halos (see FM 2012 for explicit details) and in fact in all cases the need for the DM fit occurs at radii > R (see also Begeman etal 1991). The RC discovery of DM as needed to fit the inner parts of galaxy RCs would directly falsify MOND and so far, this has not been observed for well over an obtained sample of 1000 RCs for spiral galaxies. In addition, the detailed 2D velocity field data of Beauvais and Bothun (1999) reveals that many pure disk systems have non-circular motions which are likely a response to local variations in baryonic mass density and not dominance by an extended DM halo (which would serve to smooth out these motions).

Finally there is the curious case of the galaxy UGC 128, a LSMD galaxy with a very large mass (high circular velocity). Figure 4 shows the conventional DM + halo fit to the RC:

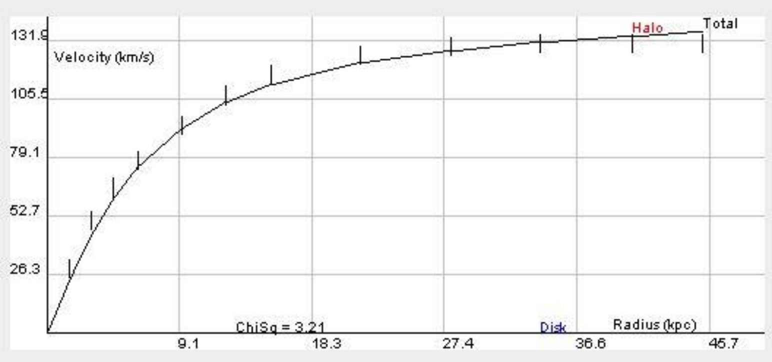

**Figure 4: Rotation Curve for UGC 128 fit with only DM Halo and no contribution from the baryonic mass components in the galaxy (e.g. stars and gas)**

In this fit, the baryonic component (stars and gas) is completely turned off and only the DM component is shown and it provides an excellent fit revealing that this galaxy is completely DM dominated. No baryonic mass is needed in contrast to most normal galaxies whose inner parts are dominated by baryonic mass and whose DM component is in the form of a large extended halo. Cases like this are distributing because it's now possible for nature to make galaxies with similar overall properties but one of which is dominated at all radii by DM whereas others are dominated by DM in only their outer regions. But UGC 128 is in the MOND limit at all radii and, the MOND fit is also represented by the black line; either this galaxy is completely dominated by baryons or it's completely dominated by DM. In other words, UGC 128 has an extremely large mass discrepancy if its dynamics are dominated by DM.

## The Bullet Cluster

For many members of the scientific community, the discovery of the "bullet cluster", a merging cluster which has displaced the hot gas relative to the respective galaxies and the subsequent analysis by Clowe etal (2004), Markevitch etal (2004) (and many others) has cemented the death of MOND. This perception seems to be truer in the world of particle physicists than astronomers, but nonetheless the bullet cluster has become the quintessential poster for the death of MOND. It is true that in general (see Sanders 1999) the overall mass discrepancy in clusters is not as easily explained in MOND as it is for the case of individual galaxies. In round numbers, the mass discrepancy in clusters is approximately a factor of 10 and MOND fits to cluster dynamics reduces that factor down to 5, still leaving a factor of 2 mass as "missing". However, it is an unwarranted leap of logic to presume that any unseen mass must necessarily be THE cold DM required by current cosmological models: it could also be as mundane as non-luminous baryons that have collected in the potential wells of galaxy clusters. It is well known both from the WMAP observations and now the Planck observations that we already have a "missing baryon" problem in that 25-50% of the required baryons in the concordant cosmology fits cannot be accounted for in known structures.

In clusters of galaxies, the best candidate for such missing baryons is stellar debris stripped off from individual galaxies as they are repeatedly tidally interacting as the overall cluster forms. Indeed, the nearby Virgo cluster shows ample evidence for such a population of intergalactic stars (Ferguson etal 1998). Similar evidence for the presence of this material in clusters can be found in Mihos etal 2005, Gregg and West 1998, Giallongo etal 2013). For the case of the Coma cluster (Gregg and West 1998), this material manifests as a "plume-like" feature approximately 150 kpc long that emanates from the central cluster. Importantly, these features are dynamically transient and will gradually be disrupted and added to the general intracluster population of stars. As these will be smoothly distributed in the potential, that population will be extremely difficult to detect. At the moment, this population is only detected when some kind of temporary structure exists (arc, plume, etc). In addition, cool gas (below X-ray temperatures) in clusters of galaxies is extremely difficult to detect and serves as another potential mass repository. As discussed further below, reconciling the mass discrepancy in clusters of galaxies with MOND would require the detection of these intergalactic baryonic populations (a currently difficult experiment) and this may be another observational route to falsification if such additional baryonic material is not discovered.

The Bullet cluster also poses a problem for DM, as the collision velocity of its components is of order 4000 km/s. In any DM dominate hierarchical merging model, pair wise velocities are quickly damped out through structure formation. However, the bullet cluster is relatively nearby in cosmological terms and there should be ample time for these pairwise velocities to damp out. Indeed, in DM simulations of structure formation (Lee & Komatsu 2010) a pair wise velocity of order 3000 km/s at the observed redshift of the bullet cluster is extremely rare ($\sim 10^{-7}$ probability or lower – see Jounghun and Eiichiro 2010). This would suggest that a) the bullet cluster is unique (and we found it?), b) the two components of the cluster in fact are not gravitationally bound (the DM solution requires they are bound), c) the attractive force between the DM particles in this region has somehow been greatly enhanced, or d) something else. As shown in Angus & McGaugh (2008) the bullet cluster configuration can occur naturally in MOND under the caveat that the two components have been falling towards each other for a substantial period of time (many billions of years)

Since the bullet cluster was discovered, similar examples of high relative velocity merging (dissociative) clusters have been observed (Markevitch etal 2005; Bradac etal 2008;). The 2012 discovery by Dawson etal of the "Musket Ball" cluster may provide an analog to what the Bullet cluster well be like, several dynamical timescales later. This system also has the lowest merging velocity of the other bullet-like systems, possibly indicating that the relative velocity damps out as the system dynamically ages, which would be expected if there is an eventual merger from a now bound state of two clusters. All of these systems reveal displacements and separation of the DM and baryonic components, consistent with the original view of the bullet cluster and the subsequent claims of its strong inconsistency with MOND. On the other hand, as discussed below, these systems are also a challenge to ΛCDM precisely because of their observed high merging velocities relatively late in the evolution of structure (see also Lee and Komatsu 2010).

## VI. Galactic Scaling predictions from MOND

In terms of the properties and structures of galaxies MOND actually gets many more details correct than DM. This is an important point to repeatedly bear in mind. Yes at scales larger than galaxies (see more below), MOND runs into a set of difficulties that are better addressed with DM, but on small scales the situation is reversed. In fact, the existence of flat RCs in MOND is the most direct and simple consequence of the reformulation of Newton's second law. Under MOND the expression for centripetal acceleration becomes

$$\frac{{V_c}^2}{R} = \sqrt{\frac{GMa_o}{R^2}} \rightarrow V_c^4 = a_o GM$$

This scaling makes two simple predictions: a) the circular velocities of galaxies scale only with their total mass (baryonic mass) and not with radius, hence **flat RCs are directly predicted** from MOND; b) total galaxy masses scale as the circular velocity to the fourth power.

In extragalactic astronomy there is a well-known correlation between the observed luminosity of a galaxy and its observed maximum rotation velocity. This is known as the Tully-Fisher (1977) relation and is of the form $L \, \alpha \, V_c^4$. When plotted in magnitude units (Mag ~ 2.5 log L) against the log of twice the circular velocity (line width) the scaling relation is manifest as a line with slope = 4. Figure 5 shows one example of this relation for a sample of cluster spirals all at a common distance. Over the entire luminosity range (factor of 100) this scaling remains linear (with low scatter) and of the slope predicted by MOND.

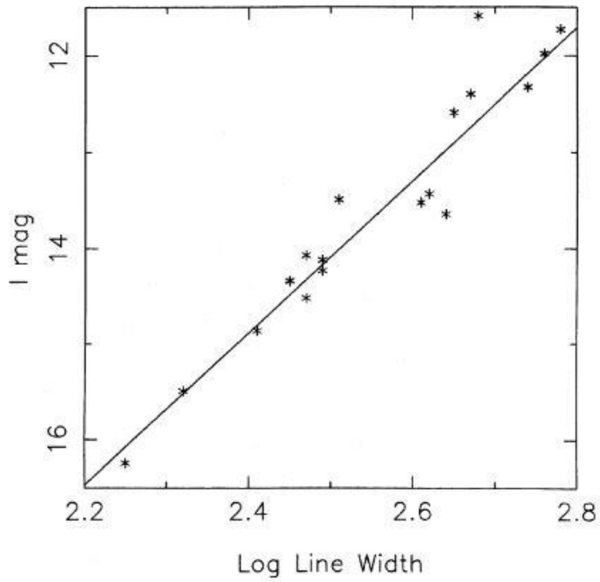

**Figure 5: Example TF relation for spiral galaxies spanning a factor of 100 in luminosity – adapted from Bothun and Mould 1987. X-axis is the log of the observed width of the 21-cm profile (essentially twice the amplitude of the flat part of the RC). Y-axis is the I-band isophotal magnitude as defined in Bothun and Mould.**

This scaling relation, of course, can also be recovered under the DM scenario but it requires special conditions as described below. However, since under this scenario galaxies are totally DM dominated there is no a priori expectation why the luminosity (optical light coming from stars) of these galaxies should rather precisely correlate with the amount of DM. To recover the slope 4 line of Figure 5 the following conspiratorial properties of galaxies is then needed.

Step 1: Virial Theorem Scaling: $M \propto V^2R$

Step 2: Replace $M$ with $L * \left(\frac{M}{L}\right)$

Step 3: Define surface brightness as: $\Sigma = \frac{L}{R^2}$

Now make the following set of assumptions:

a) $\left(\frac{M}{L}\right)$ is constant for all galaxies and let's call this constant ***B***

b) $\Sigma$ is constant for all galaxies and let's call this constant ***S***; thus $L = SR^2$

And that leads to:

$$M \propto V^2R \dashrightarrow BL \propto V^2\sqrt{\frac{L}{\Sigma}} \rightarrow L^{1/2} \propto V^2 \dashrightarrow L \propto V^4$$

Assumptions *a* and *b* are both demonstrably false when applied to real galaxies. With respect to assumption *b* we have already shown that LSDM galaxies exist and in fact, spiral galaxies exhibit a very wide range in surface brightness $(\Sigma)$ (e.g. McGaugh and Bothun 1994, McGaugh etal 1995ab, de Blok etal 1995,). In fact, these LSDM galaxies define a Tully-Fisher relation that is identical to those exhibited by normal galaxies (Sprayberry etal 1985, Chung etal 2002, Fuchs 2003). Indeed, the "baryonless" galaxy UGC 128 shown in Figure 3 (see also de Blok and McGaugh 1998) also falls on the relation despite the fact that it has essentially no "light" in the sense that no baryonic component is needed to fits its overall RC. The empirical fact that galaxies populate the same Tully-Fisher relation despite significant variations in $\Sigma$ is very hard to understand under the DM paradigm but is a natural prediction under MOND

Worse still, $\left(\frac{M}{L}\right)$ depends upon various age and metal abundance properties of the stellar population and easily varies by a factor of 2-4 between spiral galaxies. Indeed, that variation may be responsible for some of the observed scatter in Figure 5. It is also possible to use galaxy colors as an indicator of this variance such that it becomes a weak second parameter in the above scaling relation. However, this is not the main source of strangeness here. If the mass ($M$) in galaxies is dominated by DM, then this observed, tight relation between galaxy luminosity and rotational velocity becomes deeply mysterious as it demands that $M_{(DM)}/L$ is a constant. Say what? How can this be? If galaxy formation basically works via DM halos gobbling up bits of baryonic gas after recombination, how does this process know how to capture the same fractional amount of baryonic gas per halo? That is, the DM paradigm requires galaxies to have nearly the same baryonic mass fractions! In the MOND world, the slope 4 line is predicted; in the DM paradigm it would seem to arise by magic.

For most evolved galaxies, the majority of the baryonic mass is now in the form of stars, the same stars that determine galaxy luminosity (L). Hence, for these galaxies L is a good indicator of total baryonic mass. For galaxies that evolve more slowly and haven't converted most of their initial gas content into stars, then atomic and molecular hydrogen gas are also important sources of baryonic mass. In general, these more slowly evolving, gas-rich galaxies are of lower mass (factors of 10-20) than more evolved galaxies and hence have lower values of rotational velocity. McGaugh etal (2000) showed that such galaxies tend to show significantly larger scatter around the Tully-Fisher relation as the more massive galaxies suggesting the relation starts to break down at lower masses (see also Milgrom and Sanders 2007). While this possibly may indicate that the baryonic mass fraction of galaxies is a function of their total mass, this break down may be revealing something more fundamental. Indeed, Figure 6 shows the baryonic Tully-Fisher relation (BTFR) in which baryonic mass is used instead of luminosity to correlate with the observed circular velocity. Remember, the baryonic mass is the sum of stars + gas in these galaxies and the stellar light is converted to mass via the (M/L) parameter. The relation is very tight over a range of order $10^4$ in baryonic mass and the data fall along the predicted slope 4 line of MOND. This is a substantial "victory" for MOND. For the DM paradigm to successfully explain the tight relation in Figure 6 it would require that "somehow" baryons can sense the circular velocity of the DM halo they are located in and then adjust their fraction so that Figure 6 becomes observable.

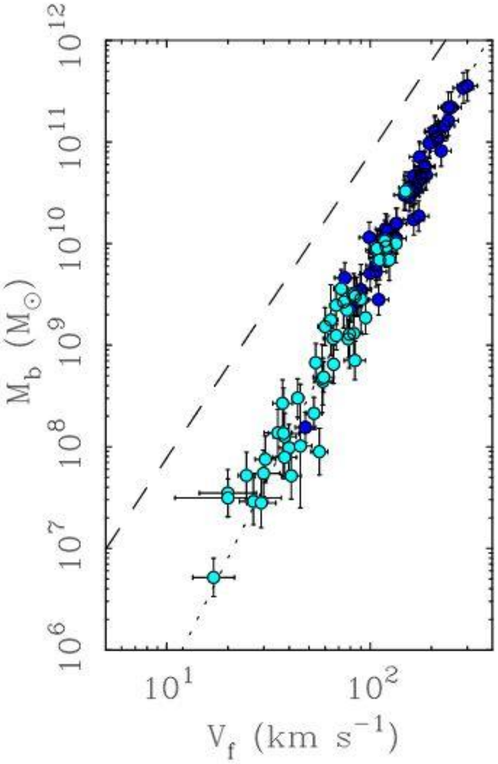

**Figure 6: Baryonic TF relation for 78 galaxies with good rotation curves and mass measurements. The dotted line has slope 4 (MOND prediction) while the offset dashed line indicates the scaling relation predicted by ΛCDM.**

However, not all galaxy rotation curves can be well fit to just the baryonic mass convolved with MOND scaling. Noteworthy is the case of NGC 3109 (Carignan etal 2013) whose rotation curve clearly demands an extended halo and a very large mass discrepancy. Randriamampandy and Carignan (2014) compile a sample of RCs that, at first glance, seem to require a variable $a_o$ for a proper MOND fit. However, those authors choose to fix stellar M/L instead of letting it vary (like it does in real galaxies). A fixed M/L necessarily drives a variation in $a_o$ due to the degeneracy of the fitting procedure between those two parameters. Most of the galaxies in the RC 2014 compilation were previously fit by Gentile et al (2007) with quite reasonable values for stellar M/L and their variations. Swaters etal (2010) also found some evidence for a small correlation between the fitted value of $a_o$ and galaxy central surface brightness (a proxy for surface mass density). Poor MOND fits to galaxies RCs can also be the result of poor distances. However, the major point here is that many galaxies, particularly those with good distances (e.g. Bottemea etal 2002), are quite well fit by a universal value of $a_o$ in combination with the observed gas and stellar baryonic mass components and that is an important achievement.

## VII. Cosmological Considerations and DM as Religion

### A brief history of cosmological models

The development of general relativity (GR) by Einstein, and its associated tenet, $E=mc^2$, serves to explain both the orbital aberration of Mercury (e.g. Le Verrier 1859) as well as providing the energy source of stars. In another instance of epicycle making, Einstein resorted to introducing a negative pressure field in the Universe to statically balance it against gravitational collapse. More specifically, the stress-energy tensor has the formal solution of:

$$P = -\rho c^2$$

This has no physical sense since the gravitational mass density of the Universe cannot be negative. A simple rearrangement of terms introduces the concept of “negative pressure”

$$-P = \rho c^2$$

This term places the Universe at a point of unstable equilibrium, any small perturbation in this static universe would cause it to either collapse or start expanding as the gravitational energy density would no longer exactly balance the negative pressure (e.g. the cosmological constant) energy density.

By 1906, galaxies became the next set of test particles in motion to be analyzed. Spectral measurements of galaxies beginning at the Lowell Observatory by VM Slipher were first published in 1913 and then subsequently by Slipher (1915, 1917, and 1921). By 1921, Slipher had published 41 galaxy spectra and all of them, except for M31 (Slipher 1913), exhibited radial motion away from the observer. Subsequent to this Le Maitre (1927) and Hubble (1929) interpreted galaxy redshift data as unmistakable evidence for universal expansion in the sense that radial velocities were directly correlated with cosmological distance. With the confirmation of the expanding universe came the realization that the Universe was not static and that the Cosmological constant could now (at least temporarily) be set to zero.

With the discovery of a very homogenous and isotropic Microwave Background (MWB), a new problem, known as the horizon problem arose as a challenge to the simple big bang expansion models of the 1970s (before there was any need for DM). Simply put, the MWB had the same physical properties in regions of the universe that have never been causally connected. This implies that the initial conditions of the Big Bang were highly homogenous, a rather unlikely occurrence. Cosmological inflation (Guth 1981) provides an elegant solution to the horizon problem by positing a short ($10^{-32}$ second) period of exponential expansion (dubbed "inflation"). During inflation, the universe would have increased in size by an enormous factor to become spatially flat. Moreover, under the inflation hypothesis, the entire universe was causally connected prior to inflation and the process of inflation itself then automatically renders the inflated universe as being homogenous.

Simple inflation makes a strong prediction: all initial curvature of space-time should have been inflated out and the large scale geometry of the Universe should be flat. A flat universe carries with it a specific constraint, namely that the sum of all possible energy densities must equal 1. In cosmological normalized units, this requirement is expressed as:

$$\Omega_{\mathrm{baryon}} + \Omega_{\mathrm{DM}} + \Omega_{\lambda} = 1$$

Since Einstein had retracted his claim of a cosmological constant, then, at this time in history (1980) we can set $\Omega_{\lambda} = 0$. Verification of the inflationary model would then occur if the other two terms sum to 1. This created an immediate problem in that the measured $\Omega_{\mathrm{baryon}}$ value was only 1-2% (e.g. Rauch etal 1997; Peacock etal 1987; Loh and Spillar 1986; Dekel 1986; Peebles 1986; Schramm 1982; Austin and King 1977). Under the inflationary paradigm simple arithmetic (1 - .01 = .99) shows that the universe is necessarily DM dominated. So by 1985, the community had arrived at some Aristotelian cosmological truth: space was flat and all of the energy density was in some DM. Given the mathematical requirement of $\Omega_{\mathrm{DM}} \sim 1$ likely meant that the DM was in the form of some mysterious particle (and this launched the discipline of astro-particle physics).

Analogous to much of the current controversy surrounding MOND (where most everyone declares it as invalid) the notion of $\Omega_{\mathrm{DM}} < 1$ was not well received or tolerated, despite observations (Davis and Peebles 1983; Aaronson etal 1986; Shaya 1986; Bothun etal 1992) which strongly showed total $\Omega \sim 0.25$, rather less than $\Omega = 1$. These dynamical measurements were primarily done by using galaxy clusters as test particles in the sense that any deviation from their expansion velocity was assumed to be caused by a gravitational perturbation of some nearby structure (e.g. another galaxy cluster) that existed in a highly clustered universe. A schematic diagram of this situation is shown below in Figure 7.

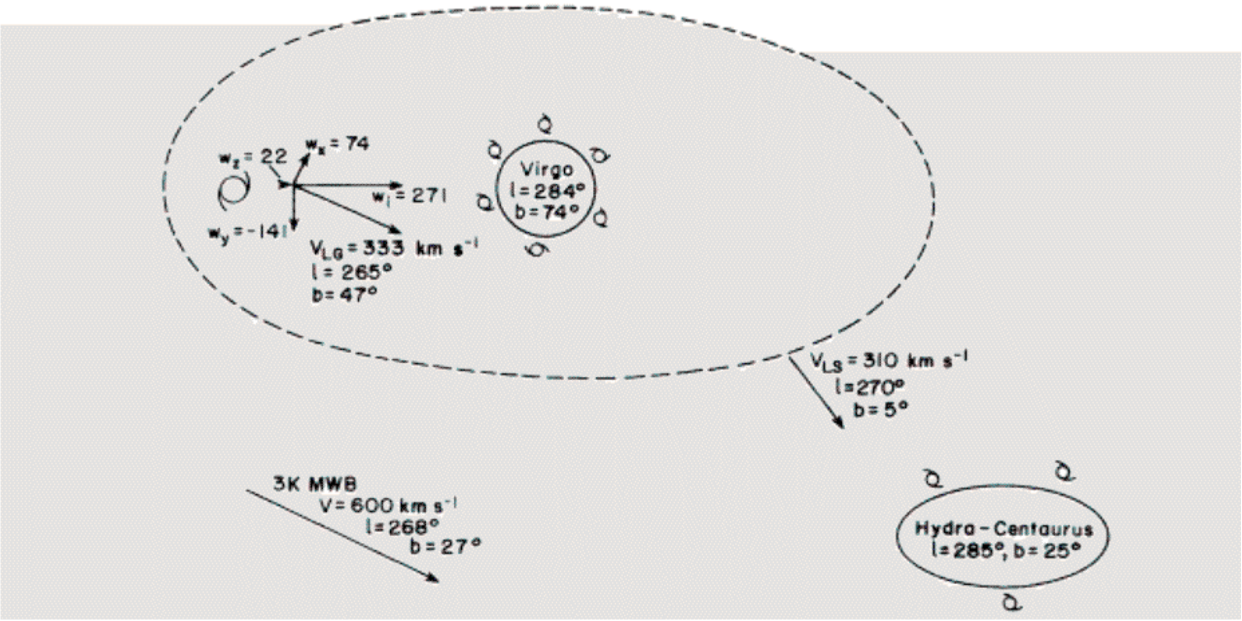

**Figure 7 – adapted from Aaronson etal 1986 which shows the motion of our galaxy infalling into the Virgo cluster and the entire Virgo cluster plus surroundings infalling into the Hydra-Centaurus supercluster. The measured infall velocities represent a gravity map from which the cosmological mass density can be derived.**

Thus the DM paradigm becomes a matter of faith as it is the necessary mechanism to save inflation since direct observations could not support $\Omega_{DM} \sim 1$. This does not seem very different from the necessary mechanism of the Ptolemaic epicycle in saving the geocentric cosmology.

Now we skip forward to the year 2001 where the concordant cosmology is emerging from various observations such as WMAP and distant supernova (e.g. Spergel etal 2003). This concordant cosmology completely sweeps away the previous cosmology. The most prominent change is the resurrection of the $\Omega_{\lambda}$ term as originally envisioned by Einstein. But, now this term is **dominant** and so we have effectively jettisoned the DM dominated Universe of the mid-1980s to a new cosmology in which $\Omega_{DM}$ has dramatically lowered to ~0.25, the exact value that the mid 1980's observations indicated. We also note that $\Omega_{baryon}$ has raised from its value of 1% in to 4.6% according to the latest analysis of the Planck MWB measurements (Planck Collaboration 2014). Thus in a period of about 30 years, the DM content of the Universe has shrunk by a factor of 4 and the baryonic content has increased by a similar factor of 4; remarkable.

## Current Cosmological Puzzles in the ΛCDM Paradigm

In the messy world of observations, we discuss 7 inconsistencies with the current theoretical paradigm. Some of these inconsistencies may well be a manifestation of gravitational forces working differently on large scales. Some of these inconsistencies might also provide new tests (see below) to falsify MOND.

1. **Large Scale Velocity Flows**: This was illustrated in Figure 7. In the **ΛCDM** model these flows are constrained to be ~ 200 km/s yet observations (Kashlinsky etal 2008) detect flows of ~1000 km/s indicating that gravitational attraction between clusters is "larger" than it is supposed to be (see also Nusser 2002).

2. **Massive Clusters of Galaxies**: Under **ΛCDM** these structures take a relatively long time to form (Mortonson etal 2011). The observed (Rosati etal 2009) cluster at z=1.4 with measured mass of 4 x $10^{14}$ $M_\odot$ clearly shows that large virialized structures can form early on in the Universe. The early formation of these structures is a serious challenge to structure formation via hierarchical gravitational merging in a **ΛCDM** universe.

3. **Galaxies in Voids**: In structure formation scenarios that are driven by gravitational hierarchical clustering, regions of galaxy under density (void) will be naturally produced. For a void size of 2.5 million light years in radius, **ΛCDM** predicts about 20 large galaxies would exist in that void (Peebles and Nusser 2010). The Milky Way is located in a region of galaxy under density and including it, there are only 2 other large galaxies (see also White and Bothun 2003) in our Local Void of similar radius. Moreover, in our local volume of radius 20 million light years there are ~600 known galaxies. The ratio of large galaxies to small galaxies is 6 times higher in this volume than what is expected from **ΛCDM** structure formation. In the case of more rapid structure formation (consistent with point 2 above) galaxies are more rapidly swept out of voids, and small galaxies are particularly prone. This would produce emptier voids at earlier times, populated primarily by a few large galaxies – in agreement with the observations.

4. **The Missing Satellite Galaxies**: This is a long standing issue in that CDM has always predicted many more small galaxies forming around large galaxies than has been observed. Moore etal 1999, for instance, predicts on order of thousands of satellite galaxies should be swarming around the Milky Way and Andromeda when in fact there are only a few dozen. Since the Moore etal treatment, new simulations that include feedback and re-ionization effects (Busha etal 2010) and more sensitive observations have lessened the severity of this discrepancy (see Bullock 2010). Since 2004 ~25 new satellite galaxies have been discovered in the Local Group, thus doubling the known population (see Willman 2010). The discovery of the "ultra-faint" population (Martin etal 2007) and the subsequent determination of their dark matter dominance (Wolfe etal 2010) suggest that more such objects may be lurking just below our detection thresholds. Most all of the recently discovered objects are devoid of atomic hydrogen so cannot be discovered in blind H I surveys (e.g. ALFALFA). Those H I surveys are more adept at finding likely tidally stripped H I which may or may not contain stars, depending on the current rate of star formation levels required to elevate their overall visibility.

Thus there now appear two possible resolutions to this issue: a) the satellite population really is there and the recent advances in detections suggest a larger population of objects (> 100 – see Bullock 2010) will be detected with more sky coverage at greater sensitivity (this is best accomplished by surveying the sky at wavelengths of ~2000 angstroms where the night sky background is at its minimum flux value (O'Connell 1987) – metal poor populations would have significant flux at those wavelengths – unfortunately there are no current instruments available for such a survey); b) the missing satellite problem is real and can only be solved by changing gravitational structure formation so that it suppresses small scale structures. While this can be done by changing CDM into W(arm)DM (Lovell etal 2011), this situation also raises questions with respect to the overall viability of **ΛCDM.**

5. **The Angular Momentum of Galaxies**: Angular momentum in forming galaxies can be acquired through tidal torques in the merger history of galaxy disks in a hierarchical formation scenario. In simulations of this process (Abadi etal 2003) angular momentum transfer occurs from the baryonic disk to the DM halo resulting in low values for specific angular momentum in the fully formed baryonic disk. This is in direct contradiction to the observations. Resolving this challenge within the **ΛCDM** paradigm essentially relies on forming disks over a longer period of time via quiescent capture of gas rather than the repeated large number of mergers required under the hierarchical formation scenario.

6. **Pure, thin disk galaxies**: Related to the angular momentum problem, disk galaxies with high rotational velocities that have no vertical structure associated with them are next to impossible to produce in any **ΛCDM** simulations (e.g. Christensen etal 2014). This is because major mergers, at any time in the galaxy formation process are disruptive and typically create some kind of vertical structure in both physical and velocity space. The simplest rearrangement of momentum takes the form of a spherically symmetric concentration of baryons in which the extended disk is embedded. This is referred to as a galactic "bulge". In these simulations, any "bulgeless" galaxies represent the quiescent tail of a distribution of merger histories for galaxies and such objects would be quite rare in the real universe. Yet within the Local Volume, such bulgeless disk galaxies represent more than half of all large galaxies (those with Vc > 150 km/s) and therefore are not rare at all (see Graham and Worley 2008; Kormendy etal 2010). The resolution of this problem is as before (see Koda etal 2009), fewer mergers and more quiescent formation of the disk via captured baryonic gas.

7. **The Missing Baryons:** The present value for $\Omega_{\text{baryon}}$ is 0.046 (Planck Collaboration 2014). However, our inventory of known baryons in the local Universe, summing over all observed stars, gas, etc., comes up short of the total. For example, Bell et al (2003) estimate that the sum of stars and cold gas is only ~ 5% of as $\Omega_{\text{baryon}}$. While this baryon shortage was emphasized in Bothun (2003) this problem is worsening as observational determinations of $\Omega_{\text{baryon}}$ have risen over the last 10 years. Bothun (2003) points out that there are two basic repositories of baryons, those baryons trapped in gravitational potentials (e.g. galaxies) and those baryons not in such potentials The discovery of LSDM galaxies suggests that some of the missing baryon population could be in the form of galaxies that are very hard to detect (see O'Neill and Bothun 2000; Read and Trentham 2005). The other alternative (perhaps more likely) is a population of intergalactic baryons in the form of a highly ionized gas. In addition, the universal baryonic mass fraction as measured by Planck is now 0.17 (see also Komatsu etal 2011) meaning that in any DM halo, 17% of the mass is in baryonic form. This expectation is actually realized in clusters of galaxies but not from the stellar component of the individual galaxies but rather from the hot intracluster gas that has been liberated from the galaxies in the cluster formation process. This provides direct evidence that baryons can be removed from DM halo potentials via the merger processes of structure formation. However, a baryonic mass fraction of 0.17 is rarely, if ever, measured in individual galaxies. Figure 8 (from FM 2012) shows the relative missing baryon fraction as a function of mass or circular velocity scale. At the low mass scale, the baryonic mass fraction approaches 1% of the global value.

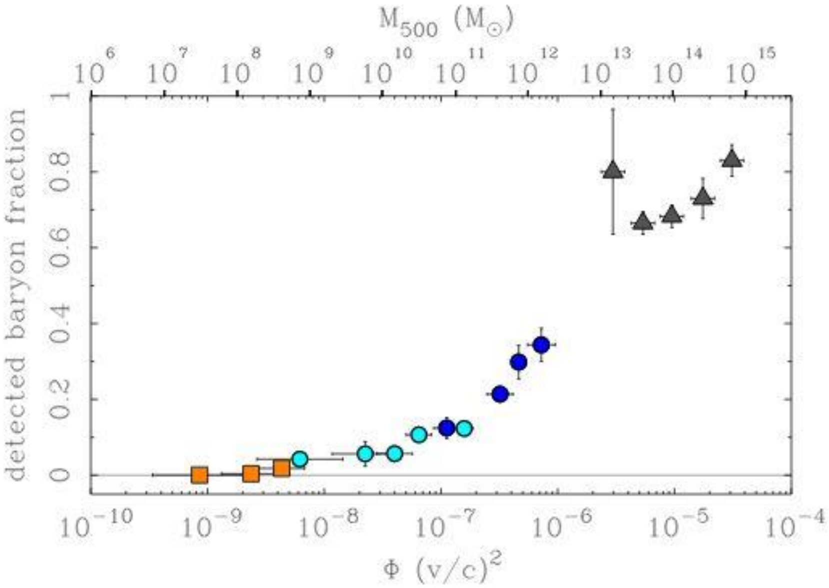

**Figure 8: Demonstration of under abundance of baryonic mass on local mass scales. At the high end (clusters of galaxies) the baryon abundance agrees with the global value (0.17) but at progressively smaller mass scales, the baryon abundance anomaly significantly worsens. Here the baryonic mass fraction (Y-axis) is the ratio of the observed baryons (stars+gas) to the dynamically determined mass. $M_{500}$ represents the dynamically determined mass contained in the radius where the average density is 500 times larger than the cosmic mean. The X-axis is a proxy for potential well depth as determined by the characteristic measured velocity dispersion.**

Hence it appears that potentials go from being least DM dominated to most DM dominated as a function of their circular velocity. This correlation is not predicted by **ΛCDM** structure formation at all (see McGaugh 2008b).

Problems 1-7 are not easily solvable with MOND but their pervasiveness strongly suggests that our standard view of hierarchical structure formation via gravitational merging under the **ΛCDM** paradigm is not very consistent with observations. Qualitatively they suggest that the merging processes driven by the "conventional gravitational force law" are not correct and/or feedback mechanisms from baryonic sources generating energy are as important as gravitational attraction. Either scenario modifies the basic operational long range attractive forces between various masses that are trying to merge. This review has not touched upon the relativistic forms of MOND which are the needed tools to apply MOND directly to structure formation (see discussion in FM 2012). Perhaps both MOND and **ΛCDM** are simply failing to get the details of large scale structure to be consistent with the plethora of observations that give rise to problems 1-7.

# VIII. New Directions/Tests to Falsify MOND

While the bullet cluster (see above) has been extensively used as the poster child for the death of MOND, the general alternative gravity paradigm will remain mostly alive until the **direct** detection DM occurs. In fact, such detection (although still indirect) may now be available to us. In the early Universe the baryons are coupled to the photons as a fluid sloshing around in evolving gravitational potentials (e.g. amplifying density enhancements which ultimately form structure). This sloshing generates an acoustic power spectrum which has a fundamental node and various harmonics. The fundamental node represents the horizon scale at the time of decoupling and is fully consistent with expectations from inflation. Figure 9 shows the angular power spectrum from WMAP and the improved spectrum from Planck.

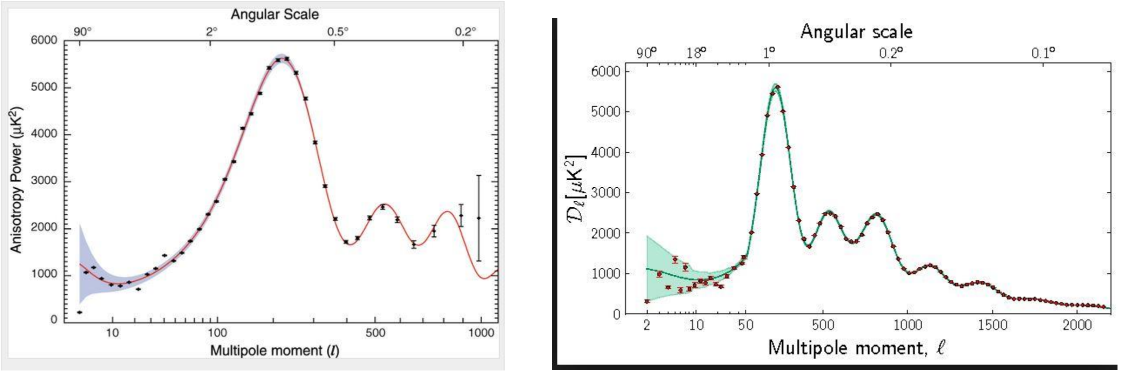

**Figure 9: Comparison of WMAP acoustic spectrum (on left) to improved S/N spectrum obtained by Planck.**

The amplitude of the second peak is driven by $\Omega_{baryon}$ and further peaks to the right represent acoustic oscillations as the photon-baryon fluid experiences perturbations. In a purely baryonic Universe, each of these successive peaks is damped relative to the previous ones. The near equality of the second and third peaks in the WMAP spectrum therefore, is inconsistent with a purely baryonic Universe. The improved S/N of Planck has greatly increased the integrity of the measurement of the third peak (and beyond) and reveals a complete symmetry between the second and third peaks. Since DM has weaker coupling to the photons, increasing the DM density (relative to the baryon density) reduces the overall amplitude of the peaks. The equal amplitudes of the second and third peaks can only occur if DM is present in the coupled photon-baryon fluid. This is strong confirmation, therefore, that DM does exist. Many would take this statement to equivalently mean that MOND is now dead.

Furthermore, the particle physics world is now vigorously investigating the plausibility that the DM particle is very heavy (> 1 TEV) and subject to decays and such decays may trigger the observed cosmic ray showers (e.g. Frampton and Glashow 1980). Much recent work (e.g. Esmaili etal 2013, Harding 2013) are quite optimistic that super-heavy DM particle decay in galactic halos is responsible for the observed cosmic-ray anisotropy. The so called WIMP miracle asserts a natural generation of DM as particle creation at that energy scale (~100 Gev) is widely observed. While the existence of DM and its ability to amplify density enhancements during the radiation dominated era may be necessary for the formation of galaxies it is not at all clear if there should be an expected relation between DM energy scale and the kinds of structures that form. Since there is an obvious relation between the number density of DM particles in a given galactic halo and the energy scale of the DM, there may be some dependence. Indeed we have already shown that the detailed properties of galaxies are not consistent with the DM halo paradigm. In this way MOND and the (particle) DM paradigm might be able to co-exist.

Recall that a very simple test to falsify MOND would be the discovery of anomalous accelerations on scales significantly above $a_o$; this has never been observed. This represents a victory for MOND and should not be easily dismissed. Below we offer three new (and difficult) observational strategies that may rule out MOND since previous attempts (described above) to falsify MOND have failed.

1. **Precision Distances to Galaxies:** Because MOND operates on a physical scale (i.e. $a_o$), all RC fits require specific distances to the target galaxies. In general there is degeneracy between M/L and distance so that the two cannot be uniquely separated. However, one galaxy, NGC 2841, with extremely high circular velocity was a good candidate for comparing the MOND required distance to a distance directly determined by Hubble Space telescope observations (see Macri etal 2001). Alas, MOND beat that test again as the HST determined distance was not sufficiently precise to rule out MOND. In general, this test will require precision distances to an accuracy of about 5% for nearby galaxies and that is extremely challenging and currently not within any of observational errors associated with these types of measurements.

2. **Mass Discrepancy in Clusters of Galaxies:** As discuss previously, MOND fails to satisfactorily resolve the observed mass discrepancy in clusters of galaxies implying, that for MOND to succeed in that environment, we need to discover more baryons. Most of the baryonic content of clusters is in the form of hot gas which radiates. However, detecting an intergalactic population of stars in the most nearby rich galaxy cluster, the Coma cluster (e.g. Gregg and West 1998), is extremely difficult, using known tracers that worked for the much nearer case of the Virgo cluster. One possibility involves the use of hostless supernova detections (the supernova progenitor star is no longer in the galaxy) as a tracer of baryonic material as a single SN traces a population of $10^{10}$ –$10^{11}$ stars. In fact, to date, we (Bothun and Robinson 2014) have discovered two hostless SN in the vicinity of the Coma cluster that may be tracers of a dynamically disrupted population of stars (see White etal 2003) that got removed from their host galaxy. If good limits can be set on the contribution of this "unseen" baryonic component, then clusters of galaxies may be the systems that ultimately do falsify MOND in a convincing manner.

3. **Dynamical tests of MOND**: In principle, if one can observe an environment in which stellar orbits transition between the MOND and Newtonian regimes, then there are strong differences in the predicted orbital evolution. For example, suppose that the vertical velocity dispersion in a disk were measured as a function of galactocentric radius from the center to the edge. In this case, the sample of stars would transition from the Newtonian regime to the MOND acceleration scale and a significant difference in the vertical velocity profile would be detected. However, this straightforward test is extremely challenging for current telescopes as there would be very little signal coming from the stellar light in the outer part of the disk from which the velocity dispersion would be measured. An easier technique, but one with less discrimination, involves measuring the velocity ellipsoid tilt angle within the meridional plane of the inner galactic disk (where there is much more signal). This tilt is different within the MOND and DM halo cases in the inner part of the Galactic disk. For the case of our Galaxy, Siebert etal 2008 and Moni Biden etal (2012) have both made measurements that are consistent with MOND predictions (McGaugh

2008a, Bienayme etal 2009). Extending this technique to the inner part of the Andromeda galaxy is now observationally possible which then doubles the sample size for this test.

## IX. Summary

Should MOND be taken seriously? Yes and no – on the one hand Milgrom's (1983) unique scaling relation and acceleration scale, $a_o$, readily explain the observed structural properties of most galaxies in terms of a baryonic generated gravitational field coupled with a modified force law. This is an impressive achievement and one in which Milgrom has clearly garnered insufficient credit. Explaining these observed structural properties in the context of the DM paradigm requires considerable fine-tuning and coincidence. On scales larger than galaxies, MOND fares less well and the acoustic power spectrum measurements now do provide good support that DM does exist. That, by itself, however, does not rule out MOND. Moreover, many of the observed large scale features are quite inconsistent with the **ΛCDM** structure formation scenarios. This point seems largely underappreciated.

FM 2012 concludes their comprehensive review with a MOND scorecard. Table 2 presents an abbreviated and annotated form of that scorecard in terms of what was discussed in this review.

Table 2: MOND Victory or Not?

| **Astrophysical System** | **MOND** | **DM** |
|---|---|---|
| Galaxy RC fits/shapes | Successful in all details and explains many details in individual RCs | Problematic in some details; requires fine tuning to account for all the observations |
| Very thin disks | Purely baryonic disks are natural in MOND | They basically don't form in the DM paradigm |
| The Tully-Fisher Relation | Naturally occurs in MOND | Requires constant baryonic mass fraction in the DM paradigm |
| Clusters of Galaxies | Cannot fully explain mass discrepancy in clusters | Existence of massive clusters at high redshift is highly problematic |
| Structure Formation | An all baryonic Universe is likely ruled out → need some DM component | Successfully predicts a highly clustered universe but misses many details; greatly over predicts the abundance of small galaxies |
| Missing Baryon Problem | Restricted to rich clusters | Pervasive |
| CMB structure/shape | Predicted the second acoustic peak a priori but not the third | Fits third peak naturally; symmetry with second peak consistent with DM present in the early Universe |

The main point shown in Table 2 is that MOND, in general, is no worse a theory to apply in most astrophysical systems than DM. Both have problems, they just have problems on different scales. Using history as a guide, it does seem unlikely that now is the epoch in which we have discovered the correct cosmological paradigms.

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