Problem Set Number 1 Solutions:

Note, R = 287 Joules kg^{-1} K^{-1}→ some of you used R in another form which was inconsistent with other units and lead to numerical problems.

In addition, most of the submitted work was not well organized or clear. I may require you to submit a PDF in the future so I can actually follow what you have done. In two cases you decided to write small that I couldn't resolve the characters easily (what is the point of that?) – you are only gonna piss off the grader by doing this.

Also, some of you made these problems much more complicated than they needed to be.

Question 1:

A) F = mg = P_o *A (A = unit cross section)

So, $m = \frac{P_o}{g}$

Pressures in millibars need to be multiplied by 100 for units in meters, kg, seconds. In general, just set g to 10.

Total mass of atmosphere = $\frac{1000x100}{10} * 4\pi R^2$

R = 6300 k m → Mass = 5 x 10^{18} kg

B) H = c_pT (per unit mass)

Total H = 5 x 10^{18} kg * 1000 *300 = 1.5 x 10^{24} (c_p for air is 1004)

C) Best way to estimate world's oceans is simply to estimate and average depth and multiply that by estimated surface area (and density). You could also use the average pressure at depth and calculate the mass the same way that you calculated the mass of the atmosphere.

Note that c_p for liquid water is about 1400. When you do the calculation you should find that H for the oceans is about 500 times larger than H for the atmosphere meaning the oceans serve as a huge heat buffer.

2)

$$\frac{dp}{p} = (-\frac{g}{RT})dz$$

Now we integrate the left hand side over p
and the right hand side over z

$$\int_{p_o}^{p} \frac{dp}{p} = \frac{-g}{R}\int_{o}^{z} \frac{dz}{T(z)}$$

well since $\int \frac{dx}{x} = ln(x)$ we have

$$ln(p) - ln(p_o) = \frac{-g}{R}\int_{o}^{z} \frac{dz}{T(z)}$$

or in terms of exponentials

$$p = p_o exp^{-\frac{g}{R}\int_{o}^{z} \frac{dz}{T(z)}}$$

To greatly simplify life we assume an *isothermal* case so
that $T(z) = T_o$ which is a constant. Now we have

$$p = p_o exp^{-\frac{g}{RT_o}\int_{o}^{z} dz}$$

let z = h so we do the integral from 0 to h to yield

$$p = p_o exp^{-\frac{gh}{RT_o}}$$

the last step is to define a scaleheight, H to be $H = \frac{RT_o}{g}$ to yield

$$p = p_o exp^{-h/H}$$

This is the functional form for pressure variations with altitude that must hold in order for hydrostatic equilibrium to exist.

For temperature of 240K nominal scale height is 7 km

Plug in the numbers to find 16,32 and 48 km for the respective heights for pressure differences of a factor of 10.

3) Many of you made this more complicated than it was:

$$N^2 = \frac{g}{T}[\Gamma_a + \Gamma]$$

$\Gamma_a = -9.8$ and your given Γ = -6.5 of a difference of 3.3

So

$$N^2 = \frac{g}{T}[3.3]$$

And solve to get N = .01 and $2\pi/N = 600$ *seconds (578 is more exact)*

4. First compute ρ at 15000 feet from the ideal gas law:

$$\rho = \frac{600 * 100}{287 * 258} = 0.81$$

The ratio of c_v to c_p = 717/1004 = .71

$$D = 0.81\left[\frac{1000}{600}\right]^{.71} = 1.17\ kg\ m^{-3}$$

5. Like problem 2 start here

$$\ln(p) - \ln(p_{0)} = \frac{-g}{R}\int_0^Z \frac{dz}{T(z)}$$

But $T(z) = T_o - (dT/dz) * z = T_o - \Gamma * z$

So that gives

$$\ln(p) - \ln(p_{0)} = \frac{-g}{R}\int_0^Z \frac{dz}{T_o - \Gamma z}$$

You can look that integral up $\int_0^x \frac{dx}{a-bx} = \frac{-1}{b} * \ln(a - bx)$

or

$\ln(p) - \ln(p_{0)} = \frac{-g}{R} * \frac{-1}{\Gamma} * (\ln(T_o - \Gamma z)$

Exponentiate that to get

$\frac{P}{P_o} = (T_o - \Gamma z)^{g/\Gamma r}$

Solving for the height (z) in the above to get 15 km for 10 K/KM

Calculate the scale height for T = 300k (8.8 km) and find 20 km as the relevant comparison height.