Problem Set Number 2 Solutions:

Seriously, make your work easier to follow!

Question 1:

First you need to assume a temperature to calculate a scale height for this problem. I choose a temperature such that H= 7.5 km

So for pressure:

$p = p_o$ exp(-z/H) = 800 millbars

Since we assume constant T then p/ρ is a constant

P = 0.8 p_0 means that ρ must also be 20% less or 1.00 kg/m^3

Question 2:

As most of you complained, this was an annoying problem on unit conversions. But it was equally annoying to me as most of you did not perform a reality check on your answer. Everyone's procedures were correct but your answers varied quite a bit depending on how you decided to calculate the cylindrical volume. These kinds of questions are not stupid and are an important part of earth system science literacy.

Procedure:

1) compute daily rainfall by mass (1.12×10^{16}) kg

2) divide that by density to get the volume of water vapor (1.12×10^{13} m^3)

3) that volume has to rain on a surface area – what fraction of the surface area of the earth is contained in this latitude range:

$Area = \int_0^{2\pi} \int_{-15}^{15} R^2 \cos\theta d\theta = 2\pi R^2 \int_{-15}^{15} \cos\theta d\theta = 2\pi R^2 * sin_{-15}^{15} = \pi r^2$

(1/4 of the total surface area of the earth is between +/- 15 degrees latitude.)

4) divide 2 by 3 and get order 10 cm per day → too high meaning the assuming that all the water vapor falls out, is pretty wrong.

Question 3:

a) mixing gives 29.5 ppt @ T = 16 C – on chart that's 1022 kg/m^3

b) adiabatic compressibility is defined as

$c_s^2 = \frac{\Delta p}{\Delta \rho}$ which is constant in this case, since the problem says the sound speed is constant

So density change is

$\rho_2 = \rho_1 + \frac{1}{c_s}(p_2 - p_1) = 1020 + \frac{99}{1500} * 100{,}000$ (convert bars to right units) = 1024.4 kg/m^3

c) basically dt/dp is too small for the initial value of parcel temperature differential to ever change; Parcel A will always be warmer.

Question 4:

a) Use ideal gas law at T = 183 and p = 1000 MB → 1.23 kg/m^3 and inverse of that is the specific volume

b) use adiabatic lapse rate for final temperature (255.5 K) and then recomputed density using 700 mb and T =255.5 → .955 kg/m^3 (big change).

c) well $\Delta H = c_p \Delta T$ and the change in temperature is 255.5 – 283 (-27.5)

change in internal energy is just 5/27of what you get above since c_v = 5/7 of c_p

d)

Considering adiabatic expansion

$$P_1 V_1^{c_p/c_v} = P_2 V_2^{c_p/c_v}$$

$$(1000\,mb)\,(1\,km^3)^{\left(\frac{1005}{717}\right)} = (700\,mb)\,V_2^{\left(\frac{1005}{717}\right)}$$

$$V_2 = 1.289\ km^3 \quad \text{or} \quad 1.2897 \times 10^9\ m^3$$

To get the mass of air work is done on

$$\rho_{700mb} = 0.95\ kg/m^3$$

$$\rho_{700mb} V_{700mb} = 1.22 \times 10^9\ kg$$

Mass of air that is "worked" on

Since adiabatic process

$$\Delta U = -\Delta W$$

Work ~~done~~ done:

$$\underset{(\text{in J})}{\Delta W} = \underset{(J/kg)}{(-\Delta U)}\,(\text{mass of air}) = 2.40 \times 10^{13}\ J$$

Problem 5:

a) First, ignore the effect of water vapor on air density. Then:

$$\rho_s = \frac{p_s}{RT_s} = \frac{10^5\ Pa}{(287.04\ J\ kg^{-1}\ K^{-1})(302\ K)}$$
$$\rho_s = 1.15\ kg\ m^{-3}$$

Using a reference pressure of 1 bar,

$$\theta_s = T_s \left(\frac{p_{ref}}{p_s}\right)^{R/c_p}$$

Since $p_s = p_{ref}$,

$$\theta_s = T_s = 302\ K$$

b)

$$\int_{T_s}^{T_t} \frac{dt}{T} = \frac{R}{c_p}\int_{p_s}^{p_t} \frac{dp}{p}$$
$$\ln\left(\frac{T_t}{T_s}\right) = \frac{R}{c_p}\ln\left(\frac{p_t}{p_s}\right)$$
$$\frac{T_t}{T_s} = \left(\frac{p_t}{p_s}\right)^{R/c_p}$$
$$T_t = (302\ K)(.125)^{2/7}$$

$$T_t = 167\ K$$

c)

Let T_{Q1}/T_{Q2} be the temps at 750 mb before/after heating.

$$\begin{aligned} Q_{latent} = Lq &= c_p(T_{Q2} - T_{Q1}) \\ &= c_p(\theta_{Q2} - \theta_{Q1})(\Pi_Q) \\ \Rightarrow \theta_{Q2} &= \theta_{Q1} + \frac{Lq}{c_p \Pi_Q} \end{aligned}$$

$c_p \Pi_Q$

If no heat is added between 750 mb and 125 mb,

$$\begin{aligned} \theta_t &= \theta_{Q2} = 302\ K\ +\ \frac{(2.5 \times 10^6\ J\ kg^{-1})(0.019)}{(1000\ kg^{-1}\ K^{-1})(.750)^{2/7})} \\ \theta_t &= 354K \end{aligned}$$

Calculating the temperature at 125 mb,

$$\begin{aligned} T_t &= \theta_t \Pi_t = (354\ K)(.125)^{2/7} \\ T_t &= 195\ K, \end{aligned}$$

$$\begin{aligned} \rho_t &= \frac{p_t}{RT_t} = \frac{(.125 \times 10^5\ Pa)}{(287.04\ J\ kg^{-1}\ K^{-1})(195\ K)} \\ \rho_t &= .223\ kg\ m^{-3} \end{aligned}$$

d)

$$\begin{aligned}
\Pi &= \left(\frac{p}{p_{ref}}\right)^{R/c_p} \\
\Rightarrow \frac{d\Pi}{dz} &= \left(\frac{p}{p_{ref}}\right)^{R/c_p-1}\left(\frac{R}{c_p p_{ref}}\right)\frac{dp}{dz} \\
&= -\left(\frac{p}{p_{ref}}\right)^{R/c_p-1}\left(\frac{R\rho g}{c_p p_{ref}}\right) \\
&= -\left(\frac{p}{p_{ref}}\right)^{R/c_p-1}\left(\frac{Rg}{c_p RT}\right)\left(\frac{p}{p_{ref}}\right) \\
&= -\Pi\frac{g}{c_p T} \\
\frac{d\Pi}{dz} &= -\frac{g}{c_p\theta}
\end{aligned}$$

Estimating z_t,

$$\begin{aligned}
\frac{(\Pi_t - \Pi_s)}{z_t} &= -\frac{g}{c_p\theta} \\
\Rightarrow z_t &= \frac{c_p(\theta_t - \theta_s)(1 - \Pi_t)}{2g} \\
&= \frac{(1000\ J\ kg^{-1}\ K^{-1})(354 + 302\ K)[1 - (.125)^{2/7}]}{2(9.8\ m\ s^{-2})} \\
z_t &= 15\ km
\end{aligned}$$

e)

$$\begin{aligned}
N^2 &= \frac{\left(\frac{g}{\theta}\right)(\theta_t - \theta_s)}{z_t} \\
&= \frac{2(9.8\ m\ s^{-2})(354 - 302\ K)}{344 + 302\ K)(15000\ m)} \\
N^2 &= 1.05 \times 10^{-4} s^{-2}
\end{aligned}$$