1)

a) The hydrostatic equation dictates that the net forces on a column of air are zero, or that the pressures are equal and opposite to the gravitation force:

$$\Delta p \cdot A = p_{bot} \cdot A - p_{top} \cdot A = mg$$

The column extends to the "top" of the atmosphere, beyond which there is no downward pressure ($p_{top} = 0$) and enforcing the unit area, $A = 1$, so the above equation becomes

$$p_{bot} = mg$$

so,

$$\boxed{m = \frac{p_{bot}}{g}}.$$

b) $p_0 = 1000\, mbar = 10^5 Pa$, $g = 9.8\, m/s^2$, $R_E = 6300\, km = 6.3 \times 10^6\, m$, $A = 4\pi R_E^2$, so the total mass of the atmosphere is

$$\begin{aligned} m &= \frac{p_0}{g} \cdot A \\ &= \frac{10^5 kg \frac{m}{s^2} \frac{1}{m^2} \cdot 4\pi \left(6.3 \times 10^6\, m\right)^2}{9.8\, \frac{m}{s^2}} \\ &= \boxed{5.1 \times 10^{18} kg} \end{aligned}$$

c) The enthalpy is given as $H = c_V T$

$$\begin{aligned} H_{total} &= m_{atm} \cdot C_{v,air} \cdot T \\ &= 5.1 \times 10^{18} kg \cdot 1047 \frac{J}{kg\, K} \cdot (300K) \\ &= 1.6 \times 10^{24} J \end{aligned}$$

d) Let $\rho(z)$ be the mass density of water in the ocean and assume (poorly) that $\rho(z) = \rho \approx 10^3 \frac{kg}{m^3}$. The surface area of the Earth is $4\pi R_E^2$, where $R_E = 6.3 \times 10^6\, m$ is the radius of the Earth. The mean depth of the ocean, h, (as found from wikipedia) is $3.7 \times 10^3\, m$. Given the oceans cover ~70% of the Earth's surface, the total mass of the oceans is approximately,

$$\begin{aligned} m_{total} &= .7 \cdot 4\pi\rho \int_{R_E - h}^{R_E} r^2 dr \\ &= .7 \cdot \frac{4}{3}\pi\rho \left(R_E^3 - (R_E - h)^3\right) \\ &= .7 \cdot \frac{4}{3}\pi \left(10^3 \frac{kg}{m^3}\right) \left(\left(6.3 \times 10^6\, m\right)^3 - \left(6.3 \times 10^6\, m - 3.7 \times 10^3\, m\right)^3\right) \\ &= \boxed{1.3 \times 10^{21} kg} \end{aligned}$$

e) Now, for the oceans,

$$\begin{aligned} H &= M_{ocean} C_{p,water} T \\ &= 1.3 \times 10^{21} kg \cdot 4185.5 \frac{J}{kg\, K} \cdot (275K) \\ &= \boxed{1.5 \times 10^{27} J} \end{aligned}$$

2)

a) For a thin parcel of air, of height dz, the hydrostatic pressure equation is

$$\frac{dp}{p} = \left(-\frac{g}{R_{air}T}\right) dz.$$

Integrating both sides gives

$$ln\left(p\right) = -\frac{g}{R_{air}T}z.$$

Exponentiating both sides yields

$$\boxed{p\left(z\right) = p_0 e^{-\frac{g}{R_{air}T}z}.}$$

b) Reverse solving for z,

$$z\left(p\right) = -\frac{R_{air}T}{g} ln\left(\frac{p}{p_0}\right)$$

With $R_{air} = 8.314\frac{m^3 Pa}{K\, mol}/.029\frac{kg}{mol}$, $T = 240K$, and $p_0 = 1000mbar$,

$$\begin{aligned} z(100mbar) &= 16166\, m \\ z(10mbar) &= 32332\, m \\ z(1mbar) &= 48499\, m \end{aligned}$$

3) Many of you made this more complicated than it was:

$$N^2 = \frac{g}{T}[\,\Gamma_a + \Gamma\,]$$

$\Gamma_a = -9.8$ and your given Γ = -6.5 of a difference of 3.3

So

$$N^2 = \frac{g}{T}[\,3.3]$$

And solve to get N = .01 and $2\pi/N = 600$ *seconds (578 is more exact)*

4. First compute ρ at 15000 feet from the ideal gas law:

$$\rho = \frac{600 * 100}{287 * 258} = 0.81$$

The ratio of c_v to c_p = 717/1004 = .71

$$D = 0.81 \left[\frac{1000}{600}\right]^{.71} = 1.17\ kg\ m^{-3}$$

5. Like problem 2 start here

$$\ln(p) - \ln(p_{0)} = \frac{-g}{R}\int_0^Z \frac{dz}{T(z)}$$

But T(z) = T_o - (dT/dz) * z = T_o - Γ *z

So that gives

$$\ln(p) - \ln(p_{0)} = \frac{-g}{R}\int_0^Z \frac{dz}{T_o \;-\; \Gamma z}$$

You can look that integral up $\int_0^x \frac{dx}{a-bx} = \frac{-1}{b} * \ln(a-bx)$

or

$\ln(p) - \ln(p_{0)} = \frac{-g}{R} * \frac{-1}{\Gamma} *(\ln(T_o \;-\; \Gamma z)$

Exponentiate that to get

$\frac{P}{P_o} = (T_o \;-\; \Gamma z)^{g/\Gamma r}$

Solving for the height (z) in the above to get 15 km for 10 K/KM

Calculate the scale height for T = 300k (8.8 km) and find 20 km as the relevant comparison height.