Problem Set Number 2 Solutions:

General Comment.

Your work is generally not well organized and difficult to follow. Some of this is a natural consequence of submitting it on paper, but you should strive to be more organized.

Also, in general, since you have been hardwired to do this, your tend to use the plug and chug approach (find the right equation, plug numbers in …) rather than the more effective and more elegant, scaling approach (see Question 1 below). Strive to be eloquent.

Question 1:

Solve this problem using the scale height, which for surface temperature of 288K is about H=7.5 km.

So for pressure:

$p = p_o$ exp(-z/H) = 800 millbars

Since we assume constant T then p/ρ is a constant

P = 0.8 p_0 means that ρ must also be 20% less or 1.00 kg/m^3

Question 2:

Mostly straight forward

1) Compute daily rainfall by mass (about 10^{14} kg)

2) Divide that by density to get the volume of water vapor that has, by definition, all turned to rain (about 10^{11} kg).

e) That volume has to rain (uniformly) on a surface area – what fraction of the surface are of the earth is contained in the given latitude range:

$$Area = \int_0^{2\pi} \int_{-15}^{15} R^2 \cos\theta d\theta = 2\pi R^2 \int_{-15}^{15} \cos\theta d\theta = 2\pi R^2 * sin_{-15}^{15} = \pi r^2$$

(1/4 of the total surface area of the earth is between +/- 15 degrees latitude.)

Question 3:

a) mixing gives 29.5 ppt @ T = 16 C – on chart that's 1022 kg/m³

b) adiabatic compressibility is defined as

$c_s^2 = \frac{\Delta p}{\Delta \rho}$ which is constant in this case, since the problem says the sound speed is constant

So density change is

$\rho_2 = \rho_1 + \frac{1}{c_s}(p_2 - p_1) = 1020 + \frac{99}{1500} * 100{,}000$ (convert bars to right units) = 1024.4 kg/m^3

c) basically dt/dp is too small for the initial value of parcel temperature differential to ever change; Parcel A will always be warmer.

Question 4:

a) Use ideal gas law at T = 183 and p = 1000 MB → 1.23 kg/m^3 and inverse of that is the specific volume

b) use adiabatic lapse rate for final temperature (255.5 K) and then recomputed density using 700 mb and T =255.5 → .955 kg/m^3 (big change).

c) well $\Delta H = c_p \Delta T$ and the change in temperature is 255.5 – 283 (-27.5)

change in internal energy is just 5/27of what you get above since c_v = 5/7 of c_p

d) Well most of you didn't do the work on this question and just said that $\Delta U = -\Delta W$ and called it good. Better to show this explicitly in terms of work done.

Considering adiabatic expansion

$$P_1 V_1^{c_p/c_v} = P_2 V_2^{c_p/c_v}$$

$$(1000\,mb)\,(1\,km^3)^{\left(\frac{1005}{717}\right)} = (700\,mb)\,V_2^{\left(\frac{1005}{717}\right)}$$

$$V_2 = 1.289\ km^3 \quad \text{or} \quad 1.2897 \times 10^9\ m^3$$

To get the mass of air work is done on

$$\rho_{700mb} = 0.95\ kg/m^3$$

$$\rho_{700mb}\, V_{700mb} = 1.22 \times 10^9\ kg$$

Mass of air that is "worked" on

Since adiabatic process

$$\Delta u = -\Delta W$$

Work done:

$$\underset{(\text{in J})}{\Delta W} = \underset{(J/kg)}{(-\Delta u)}\,(\text{mass of air}) = 2.40 \times 10^{13}\ J$$

Question 5:

Problem 5:

a) First, ignore the effect of water vapor on air density. Then:

$$\rho_s = \frac{p_s}{RT_s} = \frac{10^5\ Pa}{(287.04\ J\ kg^{-1}\ K^{-1})(302\ K)}$$

$$\rho_s = 1.15\ kg\ m^{-3}$$

Using a reference pressure of 1 bar,

$$\theta_s = T_s \left(\frac{p_{ref}}{p_s}\right)^{R/c_p}$$

Since $p_s = p_{ref}$,

$$\theta_s = T_s = 302\ K$$

b)

$$\int_{T_s}^{T_t} \frac{dt}{T} = \frac{R}{c_p} \int_{p_s}^{p_t} \frac{dp}{p}$$

$$\ln\left(\frac{T_t}{T_s}\right) = \frac{R}{c_p} \ln\left(\frac{p_t}{p_s}\right)$$

$$\frac{T_t}{T_s} = \left(\frac{p_t}{p_s}\right)^{R/c_p}$$

$$T_t = (302\ K)(.125)^{2/7}$$

$$T_t = 167\ K$$

Let T_{Q1}/T_{Q2} be the temps at 750 mb before/after heating.

$$
\begin{aligned}
Q_{latent} = Lq &= c_p(T_{Q2} - T_{Q1}) \\
&= c_p(\theta_{Q2} - \theta_{Q1})(\Pi_Q) \\
\Rightarrow \theta_{Q2} &= \theta_{Q1} + \frac{Lq}{c_p\Pi_Q}
\end{aligned}
$$

$c_p\Pi_Q$

If no heat is added between 750 mb and 125 mb,

$$
\begin{aligned}
\theta_t &= \theta_{Q2} = 302\ K\ +\ \frac{(2.5 \times 10^6\ J\ kg^{-1})(0.019)}{(1000\ kg^{-1}\ K^{-1})(.750)^{2/7})} \\
\theta_t &= 354K
\end{aligned}
$$

Calculating the temperature at 125 mb,

$$
\begin{aligned}
T_t &= \theta_t\Pi_t = (354\ K)(.125)^{2/7} \\
T_t &= 195\ K,
\end{aligned}
$$

$$
\begin{aligned}
\rho_t &= \frac{p_t}{RT_t} = \frac{(.125 \times 10^5\ Pa)}{(287.04\ J\ kg^{-1}\ K^{-1})(195\ K)} \\
\rho_t &= .223\ kg\ m^{-3}
\end{aligned}
$$

d)

$$
\begin{aligned}
\Pi &= \left(\frac{p}{p_{ref}}\right)^{R/c_p} \\
\Rightarrow \frac{d\Pi}{dz} &= \left(\frac{p}{p_{ref}}\right)^{R/c_p-1}\left(\frac{R}{c_p p_{ref}}\right)\frac{dp}{dz} \\
&= -\left(\frac{p}{p_{ref}}\right)^{R/c_p-1}\left(\frac{R\rho g}{c_p p_{ref}}\right) \\
&= -\left(\frac{p}{p_{ref}}\right)^{R/c_p-1}\left(\frac{Rg}{c_p RT}\right)\left(\frac{p}{p_{ref}}\right) \\
&= -\Pi\frac{g}{c_p T} \\
\frac{d\Pi}{dz} &= -\frac{g}{c_p\theta}
\end{aligned}
$$

Estimating z_t,

$$
\begin{aligned}
\frac{(\Pi_t - \Pi_s)}{z_t} &= -\frac{g}{c_p\theta} \\
\Rightarrow z_t &= \frac{c_p(\theta_t - \theta_s)(1 - \Pi_t)}{2g} \\
&= \frac{(1000\ J\ kg^{-1}\ K^{-1})(354 + 302\ K)[1 - (.125)^{2/7}]}{2(9.8\ m\ s^{-2})} \\
z_t &= 15\ km
\end{aligned}
$$

e)

$$
\begin{aligned}
N^2 &= \frac{\left(\frac{g}{\theta}\right)(\theta_t - \theta_s)}{z_t} \\
&= \frac{2(9.8\ m\ s^{-2})(354 - 302\ K)}{344 + 302\ K)(15000\ m)} \\
N^2 &= 1.05 \times 10^{-4} s^{-2}
\end{aligned}
$$