391 W15

(L8.1)

1) What are you getting from the course?

a) Error analysis
b) Scientific computation
c) pack a lot of info in to small
d)

2)

2) Hw questions?

(see notes)

~~Fig~~ WCS

~~L8.3~~ ~~L8.4~~ L8.2

Lec_8_discrete_eqs. H) & I)

were looking at

$$\text{amp}(c_n) = \sqrt{(\text{real}(c_n))^2 + (\text{imag}(c_n))^2}$$

and we typically ignore

$$\text{phase}(c_n) = \arctan \frac{\text{imag}(c_n)}{\text{real}(c_n)}$$

3)

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3) FFT properties

a) linearity if $h(t) = \alpha f(t) + \beta g(t)$

then $\hat{h}(w) = \alpha \hat{f}(w) + \beta \hat{g}(w)$

b) translation if $h(x) = f(x - x_0)$

then $\hat{h}(k) = e^{-ikx_0} \hat{f}(k)$

note k is equiv to w for spatial data, units are m^{-1}

phase shift

Phys 391 Lec 15

L8.45

(1) differentiation of fourier

$$\nabla \times \vec{E}(x,y,z,t) = -\mu \frac{\partial}{\partial t} \vec{H}(x,y,z,t)$$

$$+ \quad \nabla \times \mu_0 \vec{H}(x,y,z,t) = \mu_0 \vec{J} = \mu_0 \sigma \vec{E}(x,y,z,t)$$

let $\hat{\vec{E}}$ = FT of $\vec{E}$ w.r.t. both time & space

Similarly $\hat{\vec{H}}$ = FT of $\vec{H}$

then $\hat{\vec{E}} = \vec{E}_0 e^{-i(\omega t + \vec{x}\cdot\vec{k})}$ $\quad \vec{x}\cdot\vec{k} = x\hat{x} + y\hat{y} + z\hat{z}$

plug into above

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \hat{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)$$

$$+ \hat{y}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)$$

$$+ \hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$\Rightarrow$ relating k comp. to $\hat{E}$ comp. $\Rightarrow$ $-ik + ik_z$

(eg $\hat{x}$) $\quad -ik_y E_z + ik_z E_y = +i\omega\mu H_x$

etc

Part 3 of 4

L8.4

c) differentiation

$$h(t) = \left(\frac{\partial}{\partial t}\right) f(t)$$

$$\vec{h}(\omega) = i\omega \vec{f}(\omega)$$

further $h(t) = \frac{\partial^n}{\partial t^n} f(t) \Rightarrow$

$$\vec{h}(\omega) = (i\omega)^n \vec{f}(\omega)$$

eg. Maxwell eqns say

$$\nabla \times \vec{E}(t) = -\mu \frac{\partial \vec{H}(t)}{\partial t} \qquad (\vec{H} = \mu \vec{B})$$

+ $$\nabla \times \mu \vec{H}(t) = \mu \vec{J}(t) + \mu\epsilon \frac{\partial \vec{E}(t)}{\partial t}$$

for small ω, can treat top eqn

* by changing top

$$\vec{E}(\omega) = FT\ \vec{E}(t)$$
$$\vec{H}(\omega) = FT\ \vec{H}(t)$$

$$\nabla \times \vec{E}(\omega) = \mu(i\omega)\vec{H}(\omega)$$

show Ramp + Middle = "convolved with"

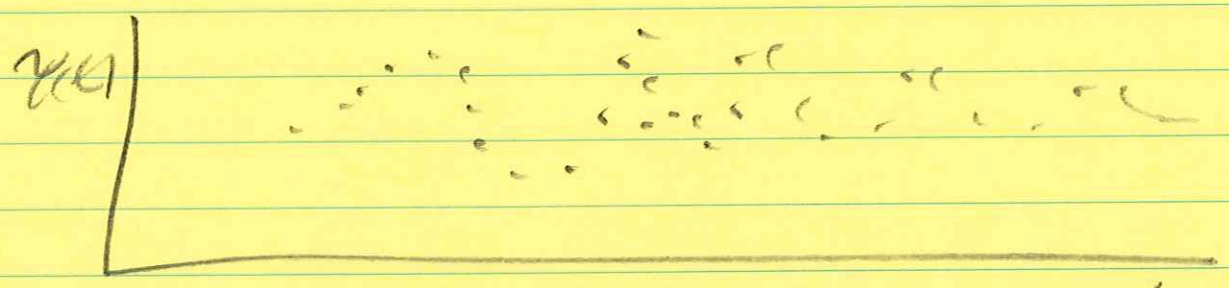
d) Convolution $h(t) = f(t) \otimes g(t)$

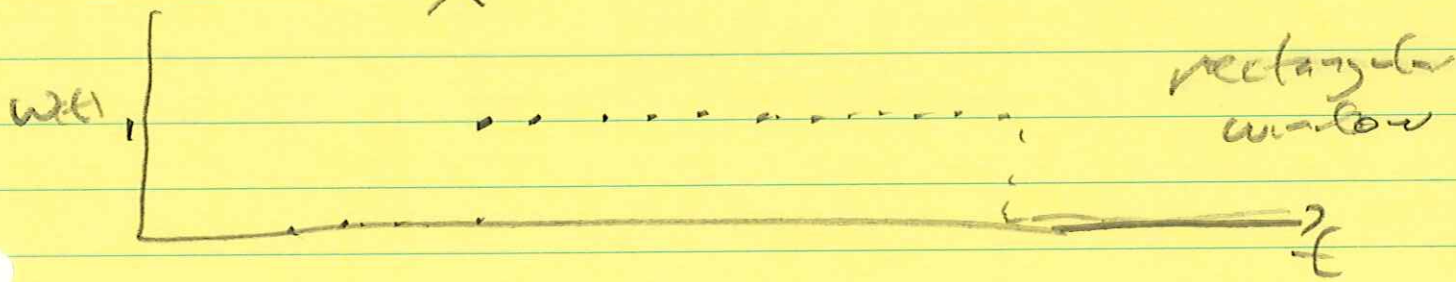
$$\hookrightarrow \equiv \int_{-\infty}^{\infty} f(t_0 - t)\, g(t)\, dt$$

then $\hat{h}(\omega) = \hat{f}(\omega) \cdot \hat{g}(\omega)$

L 8.5

Which leads to windowing
we really have this



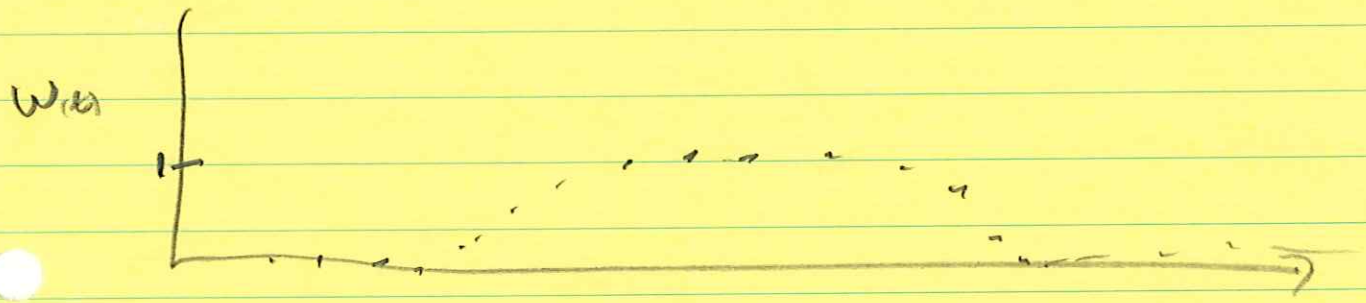


so that our $\hat{y}(\omega)$ =

$\hat{y}(\omega) \cdot \hat{w}(\omega)$ via convolution

we really want $\hat{y}(\omega)$ so we use
various windows

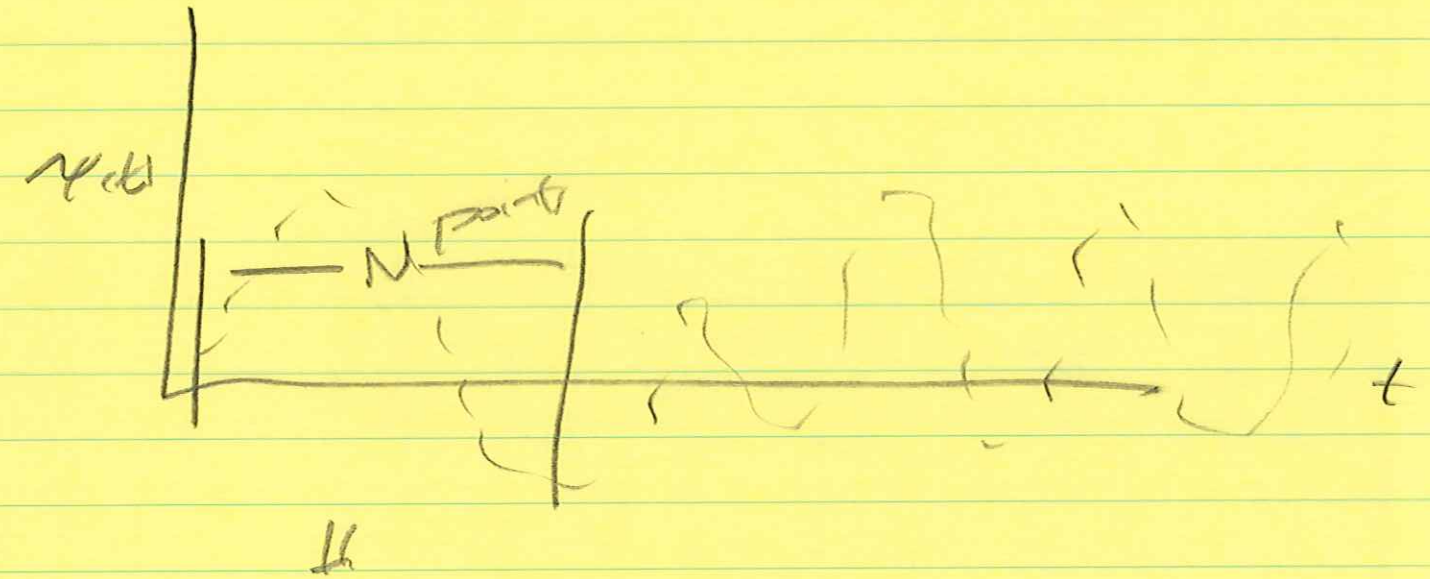
eg. cos² bell

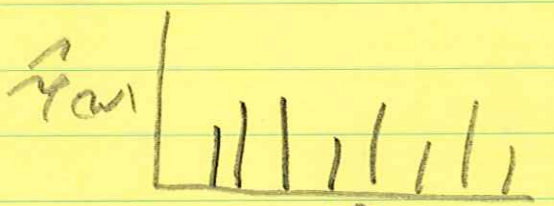


certain windows give better freq res. or
amp resolution

(L8.6)

Finally talk about sliding window





N determines spacing
high N ⇒ narrower W_f
or Δf between

But, if signal freq. content is changing with time ("non-stationarity"), then high N decreases temporal resolution

~~Lec 8 - dtz 5~~

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