PHYS 391
HW #2

4.12

The idea is that the spread in the first six measurements is enough to figure out σ_θ which can then be used as an estimate for the uncertainty in a single measurement

$$\delta\theta = \sigma_\theta$$

I would use the sample variance

$$S^2 = \frac{1}{N-1} \sum (x_i - \bar{x})^2$$

$$= 0.02 \quad \text{(from MATLAB)}$$

or the standard deviation

$$\sigma = \sqrt{S^2} = 0.14$$

So the best estimate for the uncertainty is

$$\delta\theta = \sigma_\theta = 0.14 \text{ degrees}$$

Phys 391
HW #2

4.18)

If $\sigma_u = 10\ m/s$ we can find the uncertainty on $\bar{u}$ as $\sigma_{\bar{u}} = \sigma_u/\sqrt{N}$

a) $\sigma_{\bar{u}} = 3\ m/s = \frac{\sigma_u}{\sqrt{N}} \Rightarrow N = \left(\frac{\sigma_u}{\sigma_{\bar{u}}}\right)^2$

$N = \left(\frac{10\ m/s}{3\ m/s}\right)^2 = 11.1 \rightarrow$ 11 (about 3 m/s)
12 (at least 3 m/s)

b) $N = \left(\frac{10}{0.5}\right)^2 = 400$

4.22) a)

ℓ (m)	T (s)	g (m/s^2)	
0.573	1.521	9.778	extra digit
0.611	1.567	9.823	to avoid
0.732	1.718	9.791	rounding
0.837	1.835	9.813	errors
0.950	1.952	9.843	

b) $g = 4\pi^2 \ell / T^2$ $\qquad \frac{\delta g}{g} = \sqrt{\left(\frac{\delta \ell}{\ell}\right)^2 + 4\left(\frac{\delta T}{T}\right)^2}$

$= \sqrt{(0.003)^2 + 4(0.002)^2}$

$= 0.005 \rightarrow 0.5\%$ Relative

c)

Entering these in my calculator gives

Mean: $9.8097 \rightarrow 9.810\ m/s^2$

SD: 0.0258 (using sample variance w/ N-1)

Relative variation of each measurement

$$\frac{0.026}{9.810} = 0.26\%$$

$\Rightarrow$ The variation is a bit less than the estimate of the measurement uncertainty in b)

Errors slightly overestimated

d)

$$SDOM = \sigma_x/\sqrt{N} = \frac{0.0258}{\sqrt{5}} = 0.0115$$

Now apply significant figures

$$g = (9.810 \pm 0.012)\ m/s^2$$

accepted = 9.796

$$\Delta = (0.014 \pm 0.012)\ m/s^2$$

$\Rightarrow$ 1.2σ discrepancy

4.26

a)

V:	11.2	13.4	15.1	17.7	(volts)
I:	4.67	5.46	6.28	7.22	(amps)
R:	2.398	2.454	2.404	2.452	(ohms)

I did this in MATLAB
by entering values for V & I
then R = V./I

$\bar{R} = 2.427$

$\sigma_R = 0.030$ (again N-1)

$\sigma_{\bar{R}} = \sigma_R/\sqrt{N} = 0.015$

So: $R = (2.427 \pm 0.015)$ ohms

b) It depends what is meant by '2% syst'

This will clearly be correlated for all measurements of one type (i.e. V) but may or may not be correlated to the other type (i.e. I)

Uncorrelated: $\sigma_{syst} = \sqrt{2\cdot(0.02)^2} = 2.8\%$

Correlated $\sigma_{syst} = 2\cdot(0.02) = 4.0\%$

Problem says to use uncorrelated (combine in quadrature)

So: $R = (2.427 \pm 0.015 \pm \underbrace{0.068}_{\text{doubles}}) = (2.43 \pm 0.07)\,\Omega$

Now consistent @ 1σ

5.12

$$G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

Assume $\mu = 0$ & drop the normalization

$$g(x) = e^{-x^2/2\sigma^2} \qquad g(0) = e^0 = 1$$

Find half max $g(x) = 0.5$

$$e^{-x^2/2\sigma^2} = 0.5$$

$$-x^2/2\sigma^2 = \ln(0.5) = -\ln(2)$$

$$x^2 = 2\sigma^2 \ln 2$$

$$x = \sigma\sqrt{2\ln 2}$$

Full width is twice this $\Rightarrow 2\sigma\sqrt{2\ln 2}$

$$= 2.35\sigma$$

5.21

$N = 2000$

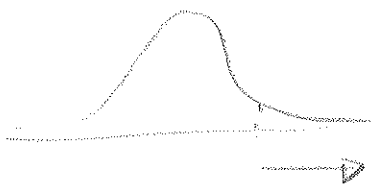
$\bar{X} = 65.5$ in

$\sigma_x = 2.5$ in

a) 5'10" = 70 in

This is 4.5 in above the mean

or $4.5/2.5 = +1.8\ \sigma$



Single-sided probability

From appendix B

$P = 0.5 - Q(1.8)$

$= 0.5 - 0.4641$

$= 3.6\%$

$2000 \times 3.6\% \equiv$ 72 people

b) Need $P = 7.2\%$ or $Q(t) = 42.8\%$

Again, Table B says $t = 1.46$

This is $+1.46\ \sigma$ from mean

$+3.65$" from mean

About 69" → 5'9" tall

5.25

There are a total of 8 measurements:

75, 75, 76, 76, 76, 79, 79, 80

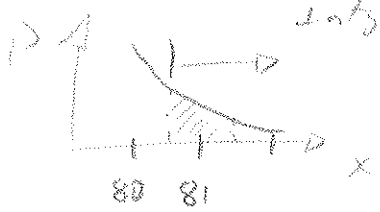
a) Using MATLAB Mean: 77.0

Std Dev: 2.0 ← $\frac{1}{N-1}$

These are the best estimates for the true (limiting) mean & sigma

$$\boxed{\begin{aligned} \mu &= 77 \\ \sigma &= 2 \end{aligned}}$$

b) Prob > 81



$$t = \frac{x-\mu}{\sigma} = \frac{80.5-77}{2}$$

$$= 1.75 \text{ sigma} \quad \text{(single sided)}$$

Prob. = 50% − 46% (Appendix B)

$$= \underline{4\%}$$

5.28/

a) Want $\frac{\delta\sigma}{\sigma} = 0.3$

$$\frac{\delta\sigma}{\sigma} = \frac{1}{\sqrt{2(N-1)}} \qquad (5.46)$$

$$2(N-1) = \frac{1}{f^2} \qquad f = \frac{\delta\sigma}{\sigma}$$

$$N = \frac{1}{2f^2} + 1$$

a) $f = 0.3 \rightarrow N = \frac{1}{2(3)^2} + 1 = 6.5 \sim 7$

b) $f = 0.1 \rightarrow N = \frac{1}{.02} + 1 = 51 \sim 50$

c) $f = 0.03 \rightarrow N = \frac{1}{2(.03)^2} + 1 = 556$

5.36/ $X_A = 13 \pm 1$

$X_B = 15 \pm 1$

a) $\Delta = |X_A - X_B| = 2.0 \pm 1.4$ ← keep extra digit!

b) $\frac{2.0}{1.4} = 1.4\sigma$ discrepancy

From table: $1 - P(\pm 1.4\sigma) = (1 - .8385)$

$= 16\%$

Consistent at the 5% level.

Note here there is no reason to assume the discrepancy sign is relevant, so a double-sided probability is entirely appropriate.