PHYS 394
HW # 3

7.1

Value	Error	Weight	$w_i x_i$
1.4	0.5	4	5.4
1.2	0.2	25	30.0
1.0	0.25	16	16.0
1.3	0.2	25	32.5

$$\Sigma w_i x_i = 83.9 \qquad x_{wav} = 1.20$$

$$\Sigma w_i = 70$$

$$\sigma_{wav} = \frac{1}{\sqrt{\Sigma w_i}} = 0.12$$

$$\boxed{x = 1.20 \pm 0.12}$$

7.7

$$x_{wav} = \frac{\Sigma w_i x_i}{\Sigma w_i} \qquad w_i = \frac{1}{\sigma_i^2} = \frac{1}{\sigma^2} \text{ (const)}$$

$$= \frac{\frac{1}{\sigma^2}\Sigma x_i}{\frac{1}{\sigma^2}\Sigma 1} = \frac{\Sigma x_i}{N}$$

$$\sigma_x^2 = \frac{1}{\Sigma w_i} = \frac{1}{\frac{1}{\sigma^2}\Sigma 1} = \frac{\sigma^2}{N}$$

$$\sigma_x = \sigma/\sqrt{N} \quad \text{as usual}$$

$$x_{wav} = \frac{\sum w_i x_i}{\sum w_i}$$

The assumption is that the weights have no uncertainty (are exact coefficients)

$$\left[\delta\left(\sum w_i x_i\right)\right]^2 = \sum \left[\delta(w_i x_i)\right]^2 \quad \text{— errors on sum add in quadrature}$$

but $\delta(w_i x_i) = w_i \delta x_i = \frac{\sigma_i}{\sigma_i^2} = \frac{1}{\sigma_i}$

So $\left[\delta\left(\sum w_i x_i\right)\right]^2 = \sum 1/\sigma_i^2 = \sum w_i$

Now the denominator is just a constant, or:

$$\frac{\delta x_{wav}}{x_{wav}} = \frac{\delta\left[\sum w_i x_i\right]}{\sum w_i x_i}$$

$$\text{So} \quad \delta x_{wav} = x_{wav} \frac{\delta\left[\sum w_i x_i\right]}{\sum w_i x_i}$$

$$= \frac{\sum w_i x_i}{\sum w_i} \frac{\sqrt{\sum w_i}}{\sum w_i x_i}$$

$$= \frac{1}{\sqrt{\sum w_i}} \quad \text{(eqn. 7.12)}$$

Prob. 4

There is clearly no "right" way to do this. My solution was:

```
function [ avg, err ] = wavg( filename )
%WAVG Summary of this function goes here
%   Detailed explanation goes here

data = dlmread(filename);

% Probably a better way to do this, but get x and y vectors
xvec = data(:,1:1);
yvec = data(:,2:2);
wt = 1./(yvec.*yvec);

err = 1/sqrt(sum(wt));
avg = sum(wt.*xvec)/sum(wt);

end
```

From this I set:

7.1: 1.20 ± 0.12

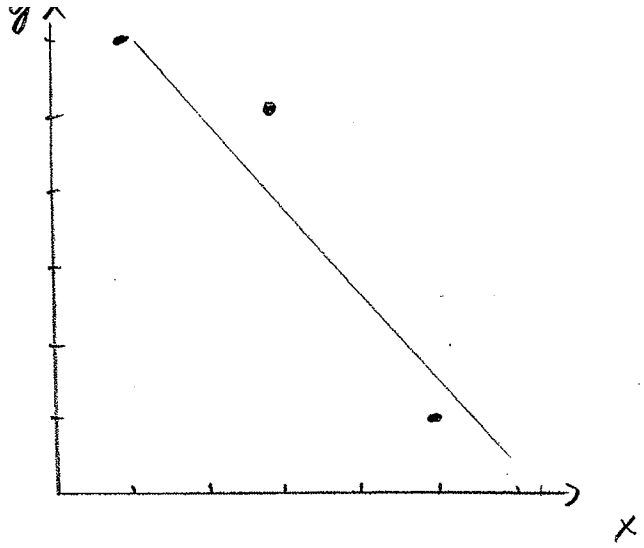
7.2: $(1968.1 \pm 0.8)\ MeV/c^2$

7.4: (500 ± 6) nm

Make sure you have the right sig. digits here!

8.1

$y = A + Bx$



x	y	x^2	xy
1	6	1	6
3	5	9	15
5	1	25	5

So $\Sigma x^2 = 35$

$\Sigma x = 9$

$\Sigma xy = 26$

$\Sigma y = 12$

$\Delta = N\Sigma x^2 - (\Sigma x)^2 = 3 \cdot 35 - 81$

$= 24$

$A = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{\Delta} = \frac{35 \cdot 12 - 9 \cdot 26}{24}$

$= 7.75$

$B = \frac{N\Sigma xy - \Sigma x \Sigma y}{\Delta} = \frac{3 \cdot 26 - 9 \cdot 12}{24}$

$= -1.25$

---

y intercept is at

$0 = A + Bx \qquad x = -A/B$

$= 6.2$

Compared to polyfit:

This gives exactly the same answer!

8.10/ I will do this by hand

x	y	σ_y	w	wx	wx^2	wxy	wy
1	2	0.5	4	4	4	8	8
2	3	0.5	4	8	16	24	12
3	2	1	1	3	9	6	2
			9	15	29	38	22

$$\Delta = \Sigma w \, \Sigma wx^2 - (\Sigma wx)^2 = 9 \cdot 29 - 15^2 = 36$$

$$A = \frac{\Sigma wx^2 \, \Sigma wy - \Sigma wx \, \Sigma wxy}{\Delta} = \frac{1}{36}(29 \cdot 22 - 15 \cdot 38) = 1.89$$

$$B = \frac{\Sigma w \, \Sigma wxy - \Sigma wx \, \Sigma wy}{\Delta} = \frac{1}{36}(9 \cdot 38 - 15 \cdot 22) = 0.33$$

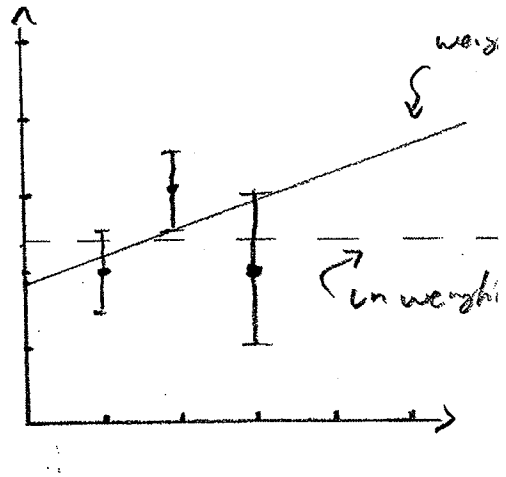
Unweighted

x	y	x^2	xy
1	2	1	2
2	3	4	6
3	2	9	6
6	7	14	14

$$\Delta = 3 \cdot 14 - 36 = 6$$

$$A = \frac{1}{6}(14 \cdot 7 - 6 \cdot 14) = 2.33$$

$$B = \frac{1}{6}(3 \cdot 14 - 6 \cdot 7) = 0$$



The 3rd point "pulls" the fit more in the unweighted case and flattens out the fit!

8.10 /

To work out the uncertainties on A & B we need

$$\sigma_A = \sqrt{\frac{\sum w x^2}{\Delta}} \qquad \sigma_B = \sqrt{\frac{\sum w}{\Delta}}$$

$$\sigma_A = \sqrt{\frac{29}{36}} = 0.90$$

$$\sigma_B = \sqrt{\frac{9}{36}} = 0.5$$

So

$$\begin{aligned} A &= 1.9 \pm 0.9 \\ B &= 0.3 \pm 0.5 \end{aligned}$$

← consistent w/ zero!

8.20

P	T
79	8
82	17
85	30
88	37
90	52

Fit to $T = A + BP$

I will just solve for A, which will be absolute zero.

Here we are doing an unweighted fit so:

$$A = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{\Delta}$$

$$\Delta = N \Sigma x^2 - (\Sigma x)^2$$

and $$\sigma_A = \sigma_y \sqrt{\frac{\Sigma x^2}{\Delta}}$$

But we don't have errors on T, so how do we find σ_y? Use variation around fit line according to eqn. 8.15

$$\sigma_y = \sqrt{\frac{1}{N-2} \Sigma (y - A - Bx)^2}$$

This time I will do this in MATLAB.

Starting with $(P, T) \rightarrow (x, y)$ I have calculated as follows:

8.20

$\Sigma x = 424$ $\quad \Sigma y = 144$

$\Sigma x^2 = 36034$ $\quad N = 5$

$\Sigma xy = 12512$

$\Rightarrow \Delta = N\Sigma x^2 - (\Sigma x)^2 = 394$

$$A = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{\Delta} = -294.9\ ^\circ K$$

I also need B to get the errors

$$B = \frac{N\Sigma xy - \Sigma x \Sigma y}{\Delta} = 3.817\ ^\circ K/mm.$$

$\Rightarrow$ A simple Matlab line fit gives the same...

$$\sigma_y = \sqrt{\frac{1}{N-2}\Sigma(y - A - Bx)^2} = 3.19\ ^\circ K$$

$\Rightarrow$ This is the uncertainty on each point in T under the Gaussian assumption (68% should be within 1σ of the line)

So $\sigma_A = \sigma_y \sqrt{\frac{\Sigma x^2}{\Delta}} = 3.19 \sqrt{\frac{36034}{394}}$

$= 30.5$

So our extrapolation for T_0 is

$$\boxed{A = (-290 \pm 30)\ ^\circ K}$$

8.25

Need to linearize by setting to:

$$z = \ln(y) = \ln(y_0) - t/\tau$$

$$= A + Bt$$

$x \to t$:	10	20	30	40	50
y:	409	304	260	192	170
$y \to \ln(y)$:	6.01..	5.72	5.56	5.26	5.14

a rounded here, but kept precision in MATLAB

Again, I used MATLAB to make this easier.

Note $\log(\ldots)$ in MATLAB is the natural log

So again:

$\Sigma x = 150$ $\quad \Sigma xy = 808.4$ $\quad N = 6$

$\Sigma x^2 = 5500$ $\quad \Sigma y = 27.68$

$$\Delta = 5000$$

$$A = 6.202 \qquad B = -0.0222$$

Mean Lifetime $\tau = -1/B$ = 45.1 seconds

Decays at $t=0$ is y_0 $\quad \ln(y_0) = A$

$y_0 = e^A$ = 493.5

Note: we don't know how many digits to speak here because we didn't work out the uncertainty!