PHYS 394
HW # 4

10.11

a) $p = 1/6, \quad n = 5 \qquad x = 0, 5$

$$P_{n,p}(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

x	P	
0:	$\frac{5!}{5!}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5$	$= 40.2\%$
1:	$\frac{5!}{1!\,4!}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4 = \left(\frac{5}{6}\right)^5$	$= 40.2\%$
2:	$\frac{5!}{2!\,3!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = 10\cdot\frac{5^3}{6^5}$	$= 16.1\%$
3:	$\frac{5!}{3!\,2!}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 = 10\cdot\frac{5^2}{6^5}$	$= 3.2\%$
4:	$\frac{5!}{4!\,1!}\left(\frac{1}{6}\right)^4\frac{5}{6} = 25\cdot\left(\frac{1}{6}\right)^5$	$= 0.3\%$
5:	$\left(\frac{1}{6}\right)^5$	$= 0.01\%$

b) 3 or more $= P(3) + P(4) + P(5) = 1 - P(0) - P(1) - P(2)$

$= 3.5\%$

c) 5 aces $= \left(\frac{1}{6}\right)^5$

Any 5 of a kind $= 6\left(\frac{1}{6}\right)^5 = 0.08\%$

10.15/ ⇒ I guess this is in the back of Taylor also...

a) $\sigma_v^2 = \overline{(v-\bar{v})^2} = \frac{1}{N}\sum_i (v_i - \bar{v})^2$

Expand $= \frac{1}{N}\sum_i (v_i^2 - 2v_i\bar{v} + \bar{v}^2)$

$= \frac{1}{N}\sum_i v_i^2 - 2\bar{v}\frac{1}{N}\sum v_i + \bar{v}^2\frac{1}{N}\sum_i 1$

$= \overline{v^2} - 2\bar{v}\bar{v} + \bar{v}^2$

$= \overline{v^2} - \bar{v}^2$

This is often written as $\sigma_x^2 = \langle x^2\rangle - \langle x\rangle^2$

b) Can assume $\bar{v} = np$

Start with Eqn. 10.5

$$(p+q)^n = \sum_v \binom{n}{v} p^v q^{n-v}$$

Take $\frac{d}{dp}$ $\quad n(p+q)^{n-1} = \sum_v \binom{n}{v} v p^{v-1} q^{n-v}$

and again $\quad n(n-1)(p+q)^{n-2} = \sum \binom{n}{v} v(v-1) p^{v-2} q^{n-v}$

For real prob. $p+q=1$

$$n(n-1) = \sum \binom{n}{v} v(v-1) p^{-2} p^v (1-p)^{n-v}$$

$$n(n-1)p^2 = \sum (v^2 - v) B_{n,p}(v)$$

$$= \overline{v^2} - \bar{v} = \overline{v^2} - np$$

or $\overline{v^2} = n(n-1)p^2 + np$

Plugging into a) $\sigma_v^2 = n(n-1)p^2 + np - (np)^2 = -np^2 + np$

$= np(1-p)$

10.14/

$$\bar{x} = \sum x\, B_{n,p}(x)$$

Bonus Solution

Starting w/ 10.5

$$(p+q)^n = \sum_x \binom{n}{x} p^x q^{n-x}$$

$$\frac{d}{dp} = \sum_x \binom{n}{x} x\, p^{x-1} q^{(n-x)} = n(p+q)^{n-1}$$

For real probabilities $p+q=1$, multiply both sides by p

$$\sum_x \binom{n}{x} x\, p^x q^{(n-x)} = np(1)^{n-1}$$

$$\sum x\, B_{n,p}(x) = np$$

10.16/

x	$B_{4,1/2}(x)$
0	$\left(\frac{1}{2}\right)^4 = 6.25\%$
1	$4\cdot\left(\frac{1}{2}\right)^4 = 25.0\%$
2	$\frac{4\cdot 3}{2}\left(\frac{1}{2}\right)^4 = 37.5\%$
3	$= B(1) = 25.0\%$
4	$= B(0) = 6.25\%$

Gauss $\mu = np = 2$

$\sigma = \sqrt{np(1-p)} = 1$

$39.9\%\, e^{-2}$	$= 5.40\%$	-0.8%
$39.9\%\, e^{-1/2}$	$= 24.2\%$	-0.8%
$39.9\%\, e^{0}$	$= 39.9\%$	$+2.4\%$
	$= 24.2\%$	-0.8%
	$= 5.4\%$	-0.8%

Good to $\sim 1\%$

$$G(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-2)^2}{2}} = (39.89\%)\, e^{-\frac{(x-2)^2}{2}}$$

10.18

18 or more heads in 25 tosses.

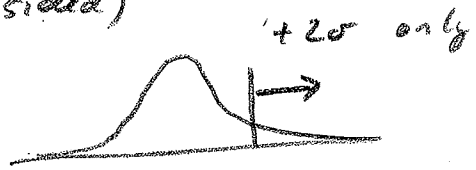
Use gaussian $\nu = np = 12.5$

$\sigma = \frac{1}{2}\sqrt{n} = 2.5$

Use $\nu \geq 17.5$

$$\frac{(17.5 - 12.5)}{2.5} = +2\sigma$$

$P(\text{above}) = [1 - P(<2\sigma)]/2$ (single sided)



$= 2.3\%$

11.3

$n = 5.0 \times 10^{19}$ $\quad p = 3.0 \times 10^{-20}$ / 5 sec.

a) Mean rate $\nu = np = 1.5$ decays / 5 sec interval

b)

$x = \nu$	$P_{1.5}(\nu) = e^{-1.5}(1.5)^\nu/\nu!$
0	22.3%
1	33.5%
2	$22.3 \cdot (1.5)^2/2 = 25.1$
3	$22.3 \cdot (1.5)^3/6 = 12.5$

c) 4 or more is $1 - \sum_0^3 P(\nu) = 6.6\%$

11.8

a) If each measurement in 15 is n_i

then $\bar{n}_{best} = \frac{1}{15}\sum n_i$ (average)

$= 145/15 = 9.67$ counts

b) I did this in MATLAB entering

$X = [7\ 11\ 10\ 7 \ldots\ ]$

mean(x) = 9.67

std(x) = 3.60

$\sqrt{\bar{n}} = 3.11$ which is within ~15%

⇒ Note, these relations are true

on average meaning that if

you either had a large number

of measurements, or did this

same thing over and over

again, the relation would asymptotically

be true.

⇒ This does justify using $\sqrt{\bar{n}}$ as

the best estimate of the uncertainty

on an individual measurement...

11.10/ Rate 20/min

Want $\frac{\delta R}{R} = 0.04$ need $\frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} = 0.04$

$N = 625$ $625/20 \cdot \text{min} = 31.25$ min.

11.20/

$N_{tot} = 225$ in 10 min $R_{tot} = \frac{225 \pm 15}{10}$

$= (22.5 \pm 1.5)$ counts/min

$N_{bgd} = 90$ in 6 min $R_b = \frac{90 \pm 9}{6}$

$= (15.0 \pm 1.6)$ counts/min

Activity $R_{tot} - R_b = (7.5 \pm 2.2)$ counts/min (Kept one extra digit here)

$= (450 \pm 130)$ counts/hr

This is a 3.5σ excess over background which would have a very low ~0.02% chance of happening from the background alone.