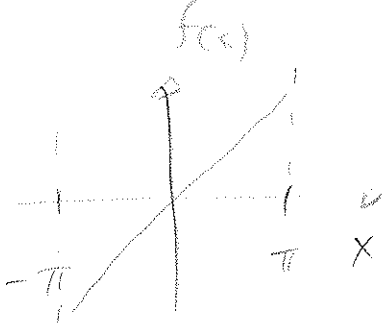
PHYS 391
HW #5

1)



So $f(x) = x$ is purely odd with no average (DC) component, so all terms $a_n = 0$. Just need to find

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin(nx)\, dx$$

Change variables w/ $u = nx$, $du = n\,dx$

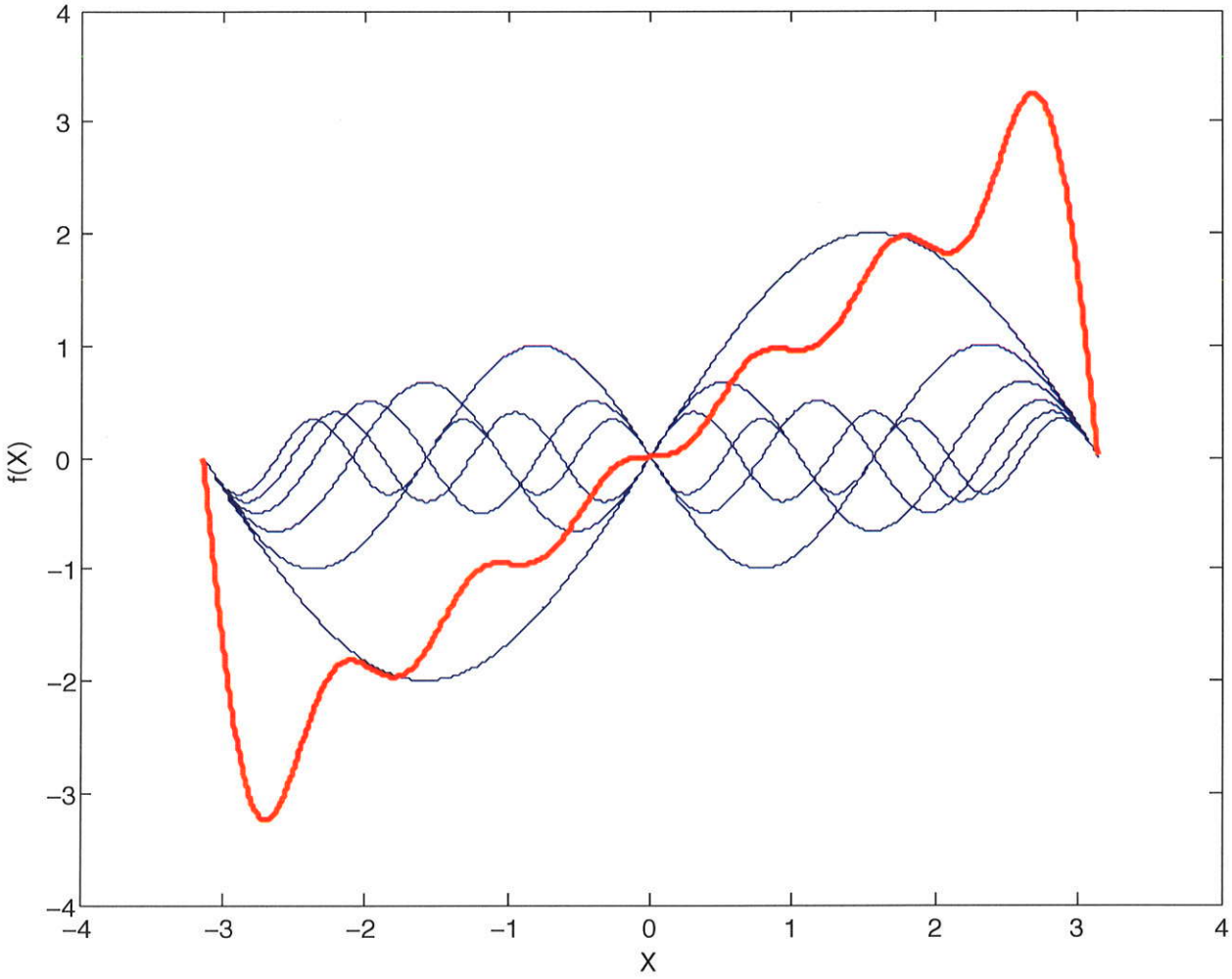
$$= \frac{1}{\pi}\int_{-n\pi}^{n\pi} \frac{u}{n} \sin u \, \frac{du}{n}$$

$$= \frac{1}{n^2\pi}\left[\sin u - u\cos u\right]_{-n\pi}^{n\pi}$$

$$= \frac{1}{n^2\pi}\left(\underbrace{2\sin(n\pi)}_{=0 \text{ for all } n} - n\pi\cos n\pi - n\pi\cos n\pi\right)$$

$$= -\frac{2}{n}\cos(n\pi) = \begin{cases} -\frac{2}{n} & n = \text{even} \\ +\frac{2}{n} & n = \text{odd} \end{cases}$$

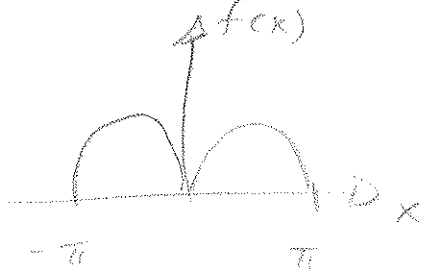
$$= (-1)^{n+1}\frac{2}{n}$$



Here I overlayed the individual terms, although just the sum is fine also.

2)

$$f(x) = \begin{cases} -\sin(x) & -\pi < x < 0 \\ \sin(x) & 0 < x < \pi \end{cases}$$



So this is purely even,

so we should only have terms a_n.

Lets do the constant term separately

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} -\sin x\,dx + \frac{1}{\pi}\int_{0}^{\pi} \sin x\,dx$$

$$= \frac{1}{\pi}\int_{0}^{-\pi} \sin x\,dx + \frac{1}{\pi}\int_{0}^{\pi} \sin x\,dx$$

$$= -\frac{1}{\pi}\left[\cos x\right]_{0}^{-\pi} - \frac{1}{\pi}\left[\cos x\right]_{0}^{\pi}$$

$$= -\frac{2}{\pi}\left[\cos x\right]_{0}^{\pi} = -\frac{2}{\pi}(-1-1) = \frac{4}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{0}^{-\pi} \sin x \cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} \sin x \cos nx\,dx$$

$$= \frac{2}{\pi}\left[\frac{\cos(1-n)x}{2(1-n)} - \frac{\cos(1+n)x}{2(1+n)}\right]_{0}^{\pi}$$

again, since integral only involves cosines, $\pm\pi$ is the same

Now look at n - odd or n - even

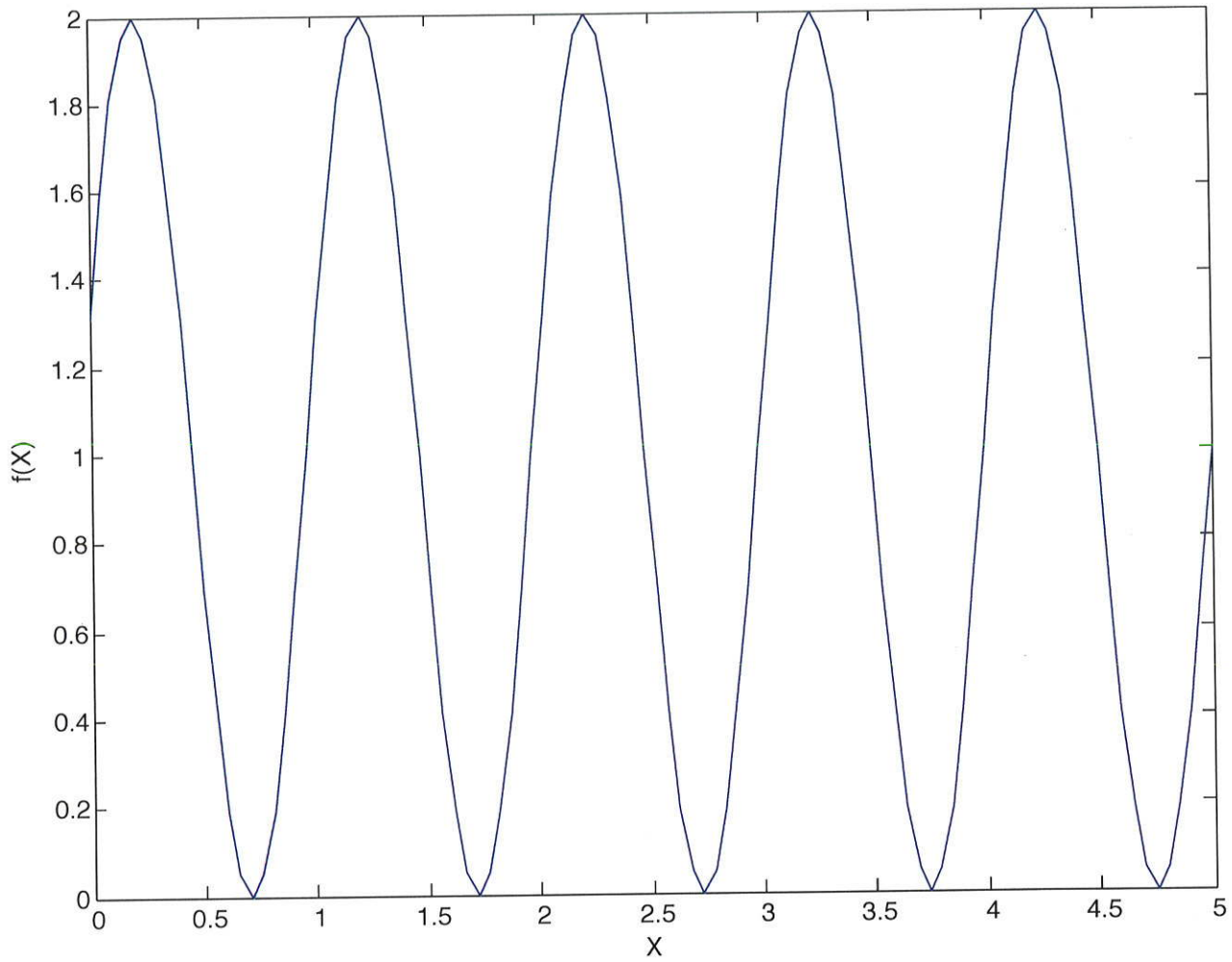
$$a_{odd} = \frac{2}{\pi}\left[\frac{1}{2(1-n)}(1-1) - \frac{1}{2(1+n)}(1-1)\right] = 0$$

$$a_{even} = \frac{2}{\pi}\left[\frac{1}{2(1-n)}(-1-1) - \frac{1}{2(1+n)}(-1-1)\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{(n-1)} + \frac{1}{(n+1)}\right] = \frac{2}{\pi}\,\frac{n+1+n-1}{n^2-1} = \frac{2}{\pi}\,\frac{2n}{n^2-1}$$

So the lowest term is $n=2$ $\quad a_2 = \frac{8}{3\pi}$

or twice the frequency of the unrectified sin(x) function.



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sample.m

```
function [ f ] = sample( A, B, nu, nus, N )
%SAMPLE Create Fake data
%   Create fake data sampled at frequency nus for N points.
%   f(x) = A sin(2pi nu x) + B

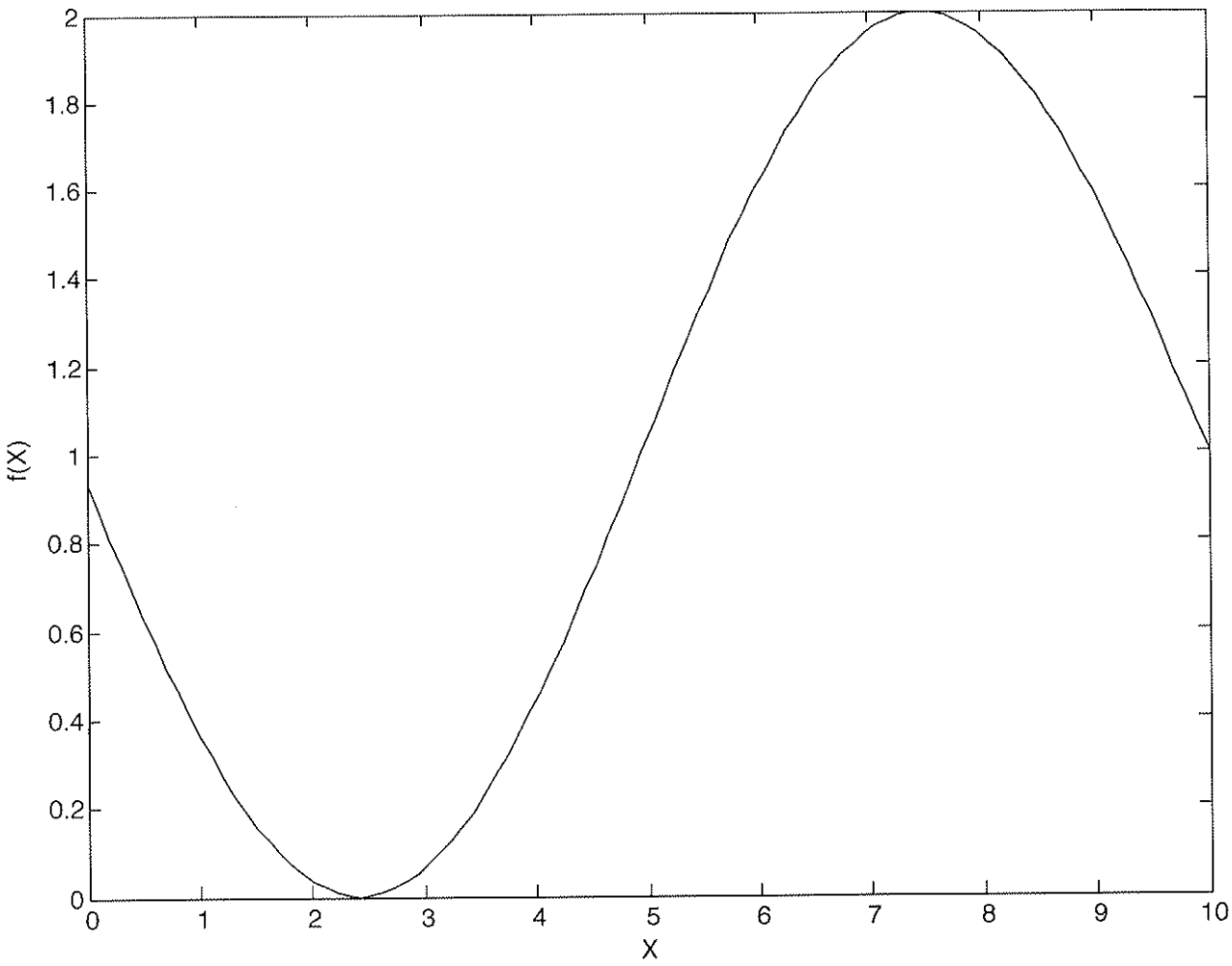
dx = 1/nus;
x = (1:1:N)/nus;
f = A * sin(2*pi*nu*x) + B;
end
```

To test, I just did

```
x = linspace(0, 5, 100)
f = sample(1, 1, 1, 20, 100)

plot(x, f)
```

4)



testsample.m

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```
N = 100;
nu = 9.9;
nus = 10;
x = linspace(0, N/nus, N);
f = sample(1, 1, nu, nus, N);
plot(x,f);
xlabel('X');
ylabel('f(X)');
```

Same sample function, but now I see clear aliasing, as my output has apparent frequency of 0.1 (1 cycle in 10)

$\Rightarrow |9.9 - 10|$

5)

```
function fftplot( data, nus)
%FFTPLOT Plot amplitude vs. frequency
%   data is an array of data sampled at nus

N = length(data);      % Determine N from data directly

Xarray = fft(data);    % Complex Coefficients
Xnorm = abs(Xarray)/N; % Normalized real amplitudes

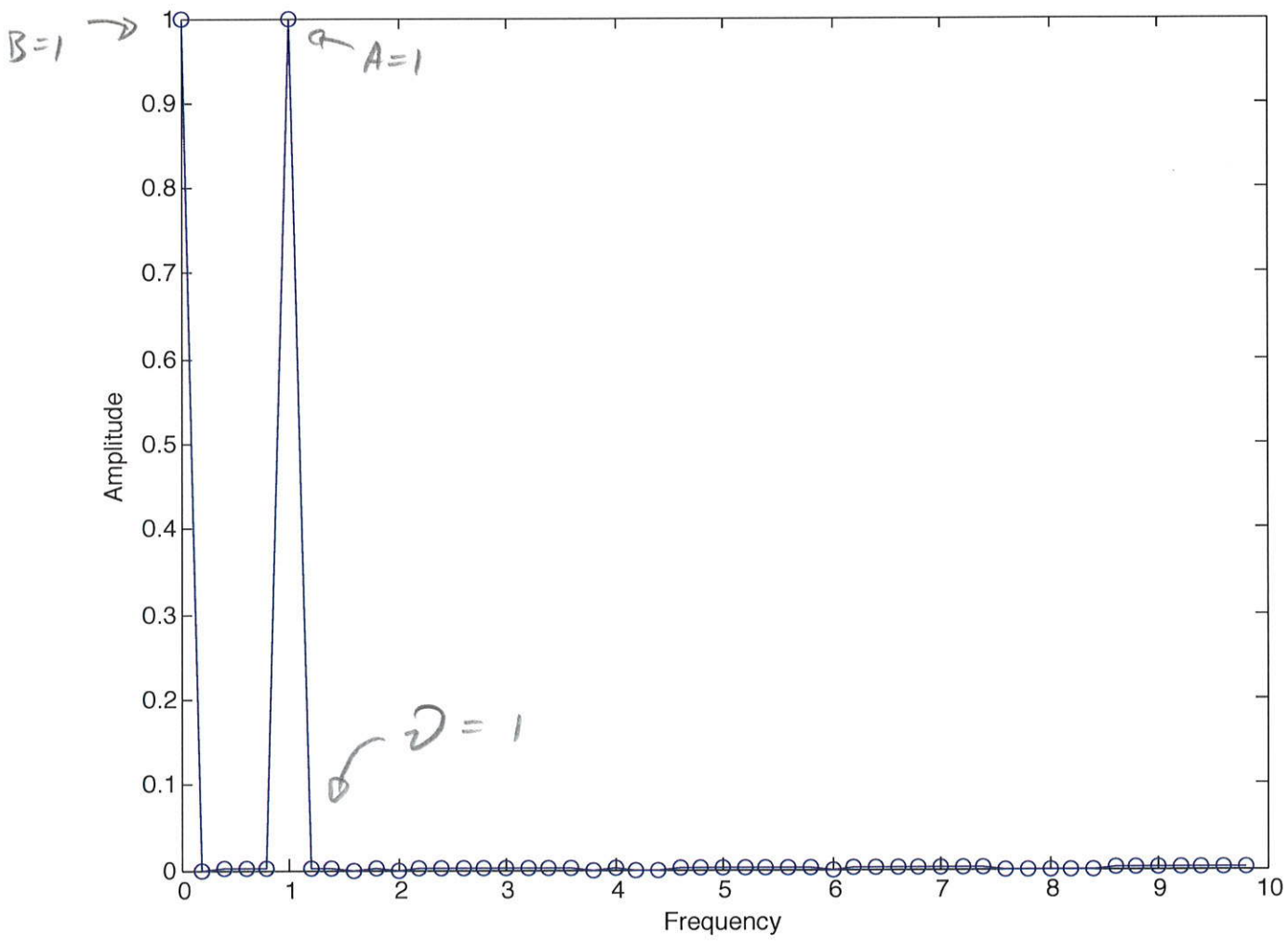
% To get amplitudes, just take first half of output and double
amp = 2*Xnorm(1:N/2);  % First element is indexed by 1
amp(1) = amp(1)/2;     % Don't want to double frequency=0 component

% Now build the frequency range this corresponds to and plot
freq = 0:nus/N:nus*(N-1)/(2*N);  % N/2 total elements
plot(freq, amp, 'marker', 'o');
xlabel('Frequency');
ylabel('Amplitude');
end
```

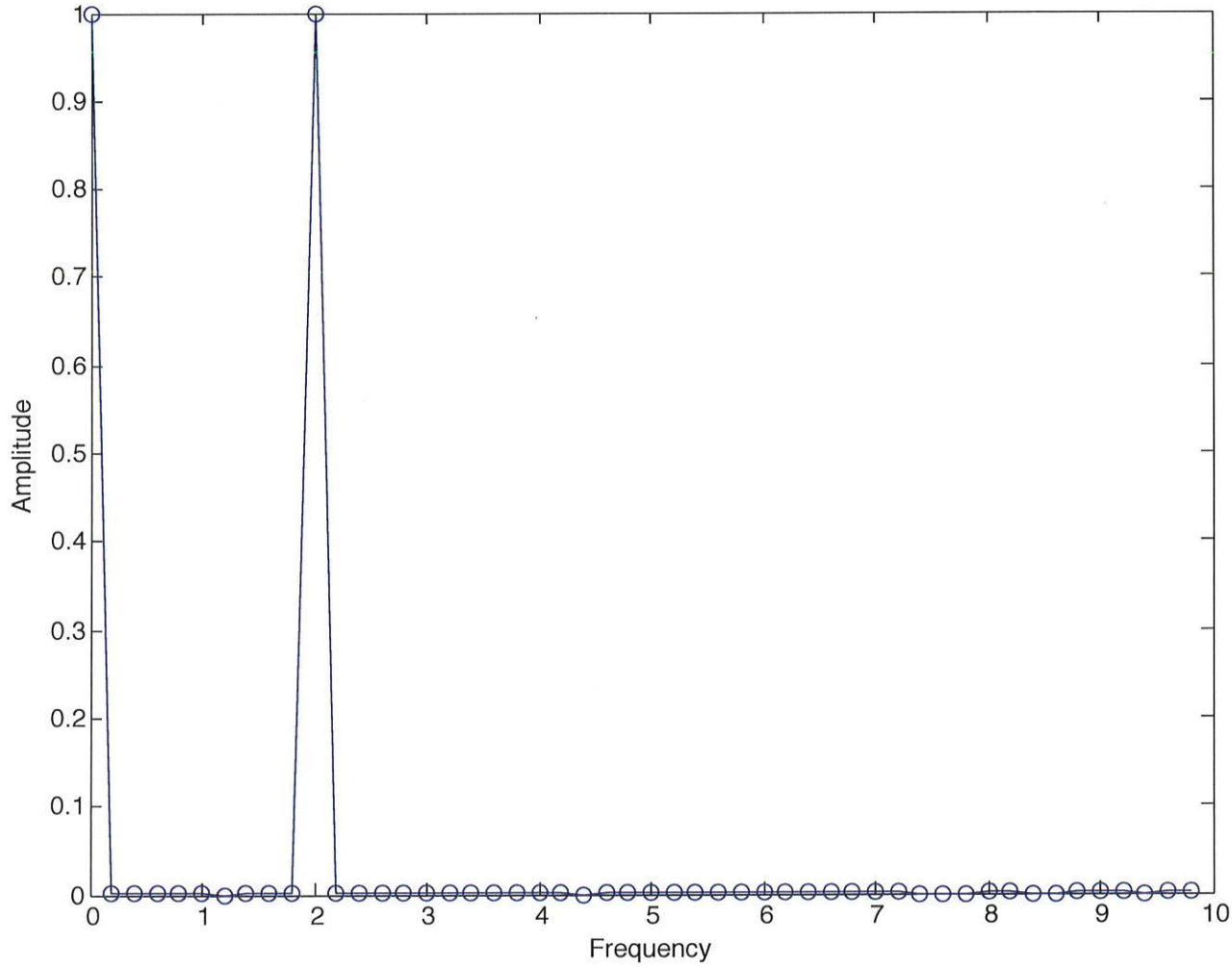
f = sample(1, 1, 1, 20, 100)

fftplot(f, 20)

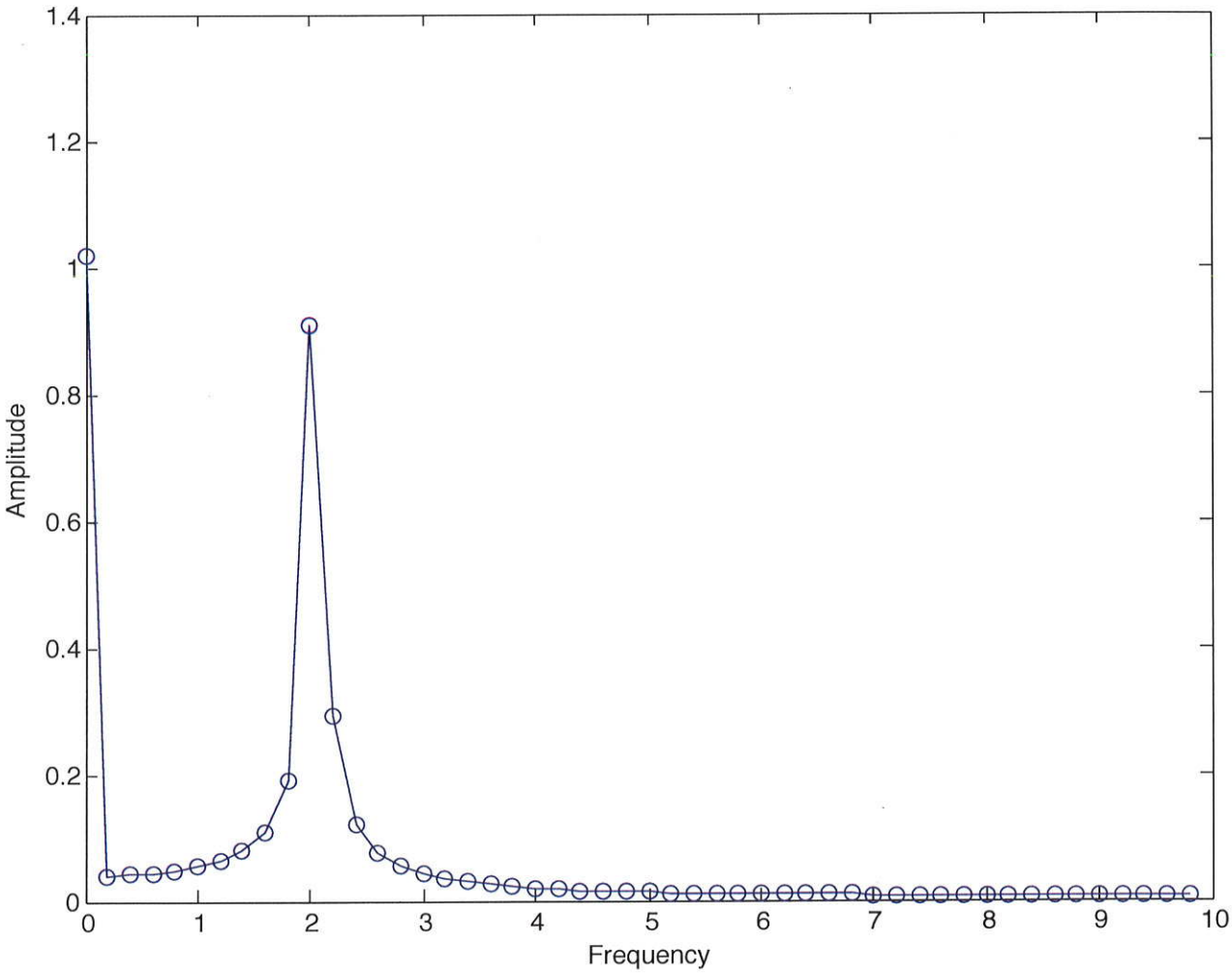
Applied to the data from 3)



6) Same thing, but with $\omega = 2$



7) Same thing but with $\partial = 2.05$. Now ∂ isn't an even multiple of $\partial_{S/N}$ and we see the result of windowing. In other words we are really taking the FT of a function ~~~~



8) Same thing w/ N = 1000. This does not change ω_s in the plot below (still cuts off at $\omega_s/2 = 10$) but it improves the frequency resolution by a factor of 10. The windowing looks narrower because the 'box' in now 10 times longer

